ADVANCED DYNAMO LABORATORY EXPERIMENTS

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ARMOUR INSTITUTE OF TECHNOLOGY

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A THESIS

PRESENTED BY

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AND M. C. VEREMIS

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INDEX.

Preface.

Knowledge of certain things is obtained either by observation and study, or by experiment. Each one of the three factors is as important as the other.

The characteristics of any piece of apparatus are found experimentally, Experimenting is a means of either checking theorectical results, or a means of arriving at new results.

Therefore, since, characteristics are necessary it follows that experimental work is very important and highly desirable.

Believing that experimental work is a necessary factor accompanying theory which otherwise would be incomplete, the authors chose a number of useful and advanced experiments for this thesis.

The authors desire to acknowledge indebtedness to Professor Freeman and to Professor Snow, both of the Armour Institute of Technology for their many suggestions, criticisms, and guidance, without which this work would be incompletee

EXPERIMENT NO. 1.

PHASE CONVERTERS.

OBJECT.

It is experimentally found that polyphase machines are more efficient and give more satisfactory results than single-phase.

There are sometimes cases where three phase line is not available and only three-phase machines are on hand. As, for instance, in operating a locomotive on a heavy grade. A three-phase induction motor is fed from a single trolley of a single-phase line construction.

It is therefore, the scope of this experiment to show how satisfactory results can be obtained by an induction squirrel cage motor as a phase converter.

DEFINITIONS.

Speaking about phase as an electrical term, we mean the distance usually in angular measure, of the base of any ordinate of an alternating wave from any chosen point on the time axis. ((A. I. E. E. rules))

When corresponding cyclic values of two sinusoidal alt. quantities of the same frequency occur at different instants, the two quantities are said to differ in phase by the angle between their nearest corresponding values; e.g., the phase angle between their nearest zeros, or their nearest positive maxima.

Single-phase, is a circuit energized by a single alternating e.m.f. supplied through two wires. The currents in these two wires counted positively outward from the source, differ in phase by 180 degrees, or a half-cycle.

Two-phase, is a combination of two circuits energized by alternating e.m.fs which differ in phase by 90 degrees, or quarter of a cycle.

Three-phase is a combination of three circuits evergized by alternating e.m.fs which differ in phase by 1/3 of a cycle, or 120 degrees.

A phase converter is machine converting A. C. of one phase to an A. C. of another phase but of the same frequency.

POLYPHASE TO SINGLE PHASE.

In general, it would not be advisable to use an induction machine in this manner to transform the number of phases, since for this purpose a number of transformers, connected in well-known ways, would be cheaper and better. In two cases, however, this arrangement may be advantageous. It can be readily shown that it is not possible by any combination of transformers to change from a polyphase system to single-phase, and have the currents of the polyphase system balanced. The energy will in all cases be supplied as single-phase energy from the generator. In general the power in single-phase circuits falls to zero four times in each cycle. The power supplied by the polyphase line must likewise fall to zero, unless some means of storing energy in the system is provided. The kinetic energy of the rotor of an induction motor, used as a phase converter, supplies a means of storing this energy. The angular velocity of the rotor is not constant, but varies during the revolution. The machine takes energy in approximately equal a-

mounts from all the phase of the supply system. When no energy is demanded by the single-phase system, the motor is being accelerated, and energy is being stored as kinetic energy in the rotor. During the parts of the cycle when the energy demand of the single-phase system is heaviest, the rotor is retarded and gives up a part of its kinetic energy.

SINGLE PHASE TO POLYPHASE.

The induction machine may also be used in the opposite manner to transform from single-phase to any polyphase system. When operating near synchronism on a single-phase system, provided the motor has a low-resistance rotor, a rotating magnetic field will be nearly constant in all positions. Hence a polyphase e.m.f. will exist at the terminals of the corresponding windings. The voltages of the polyphase windings will not be exactly balanced under load, but will be nearly so.

UNBALANCING.

When the induction motor is used as a phase converter, it always produces some unbalancing of voltages and currents, which is sometimes very large. When unbalancing is undesirable, it is usually corrected by adding to some of the phases auxiliary voltages.

APPLICATION OF ABOVE PRINCIPLES.

An example of an application where this principle might be used to advantage is in the case of a plan where it is desired to introduce motors to drive the machinery, and only single-phase energy is available. Of course, this condition might be met by the use of single-phase motors. As is well known, however, such motors are very costly, compared with polyphase motors of the same rating. Morever, in many cases there is a strong possibility that the supply will later be changed to polyphase. Under these circumstances, it might be well to install standard three-phase motors, starters, and wiring. The

three-phase supply lines could then at any future time be connected directly to the wiring, and the plant operated three-phase. To test how satisfactory results can be obtained by an induction motor as a phase converter, we proceed as follows. (See method of performance.)

PHASE SPLITTING DEVICE.

During the time when single-phase energy alone is available, it would be necessary to add a phase-splitting device as indicated in Figure. This can be constructed as shown be connecting a resistor and a reactor in series. It need only be large enough to start one of the three-phase motors unloaded. It may then be disconnected from the line, and the one motor in operation will generate such an e.m.f. as to cause the third wire to assume the proper phase relation to the other two, and form with them nearly a true three-phase system. The other motors may then be started as three-phase motors. A peculiarity of this system is that while the pull-out point of all the motors loaded, each in proportion to its

rating, would correspond to the single-phase rating of the motors, the pull-out point of any single motor, provided the other motors are lightly loaded, is practically the same as its pull-out point on a three-phase circuit. This property may be of great value in cases where motors are subject of heavy momentary overloads.

BEHAVIOR OF MACHINE AS VOLTAGE AND POWER BALANCER.

As a corollary of the use described, it will be readily seen that if an induction mac ine be operated from a polyphase line, the voltages of which are unbalanced, that it will act to restore to a certain extent the correct voltage relation. In fact, the case just described is merely an extreme case of unbalanced voltage. We may consider the system as a three-phase system in which the voltage between either of the two outside lines and the third line is indefinite; that is, it may be at the potential of either of the other two lines. By starting

the induction machine, it is caused to assume a certain definite phase relation to the other two lines.

In a similar way, if an induction machine is operated on an unbalanced circuit, it will tend to take most of its energy from the phases, the voltages of which are high, and little or none from those which are low. In fact, it may readily happen that it even returns energy to the low-voltage circuits, and of course, takes an extra amount of energy from the high-voltage circuits. Thus the machine may be acting at the same time as a motor and a transformer, taking energy from certain heavily loaded circuits and transferring it to others more lightly loaded.

RESULTS.

These theoretical assumptions agree with the experimental results obtained.

In the first part the energy supplied to each phase of the machine is 0, and 130 Watts respectively. Now, as the load is thrown on phase

B, and varied from 0 to 40 amperes, the energy supplied to the loaded side increases in proportion, while that supplied to the loaded side decreases and finally reverses sign, indicating that the loaded phase, by transformer action, acts as a generator and supplies the load.

In part number (2) the second phase is entirely disconnected from the set supply after the motor is started and loaded with a non-inductive load. The load is increased from 0 to 15 amperes supply and the transformer action induces an e.m.f. in phase B, which is cut out.

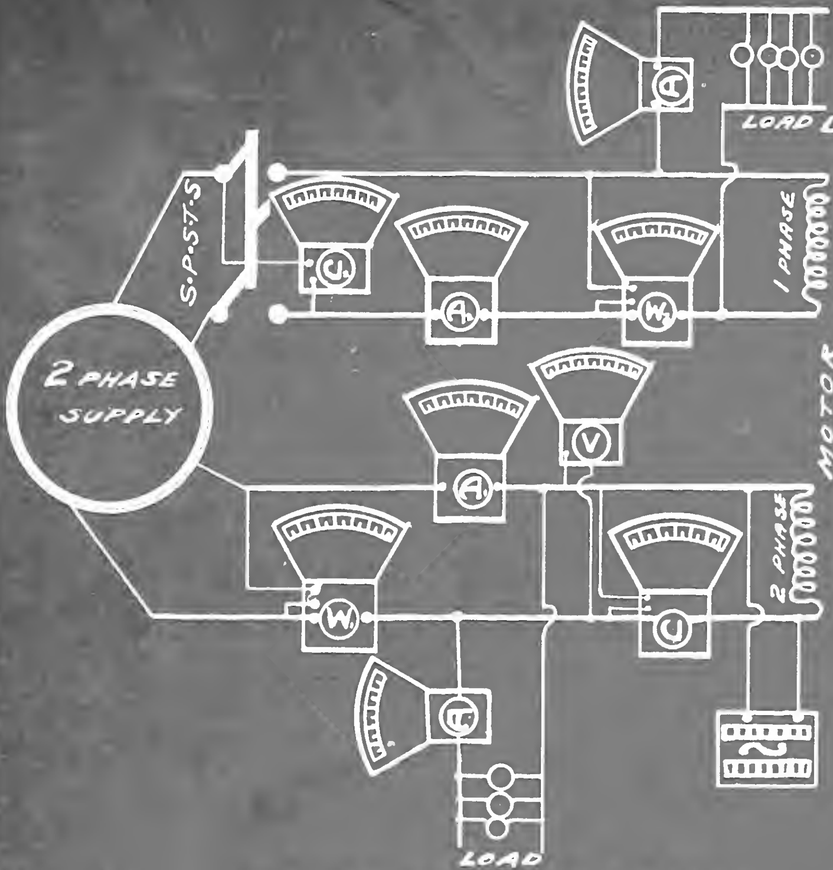
METHOD OF PERFORMANCE.

(a) Run induction machine two phase and put a lamp load on one phase. Note load and voltage on two phase lines going to motor, as well as load and voltage on lamps. Look particularly for any unbalance of load to motor.

((See scheme of connections, figure 1A))

(b) Operate motor single phase and load two phase with lamps, or other non-inductive load. Note load current and voltage for each phase of the load, phase relation between two phase voltage, as well as single phase load to motor.

((See scheme of connections figure 1B))



TEST OF A 25 ~, 10 HP, 80 VOLTS
2 PHASE
INDUCTION MOTOR

1st PART— SWITCH CLOSED LOAD L OFF
2d PART— SWITCH OPEN LOAD L ON.

TEST OF A (2) PHASE INDUCTION MOTOR
AS A PHASE CONVERTER.

MOTOR 25 , 10 HP,80 VOLTS.

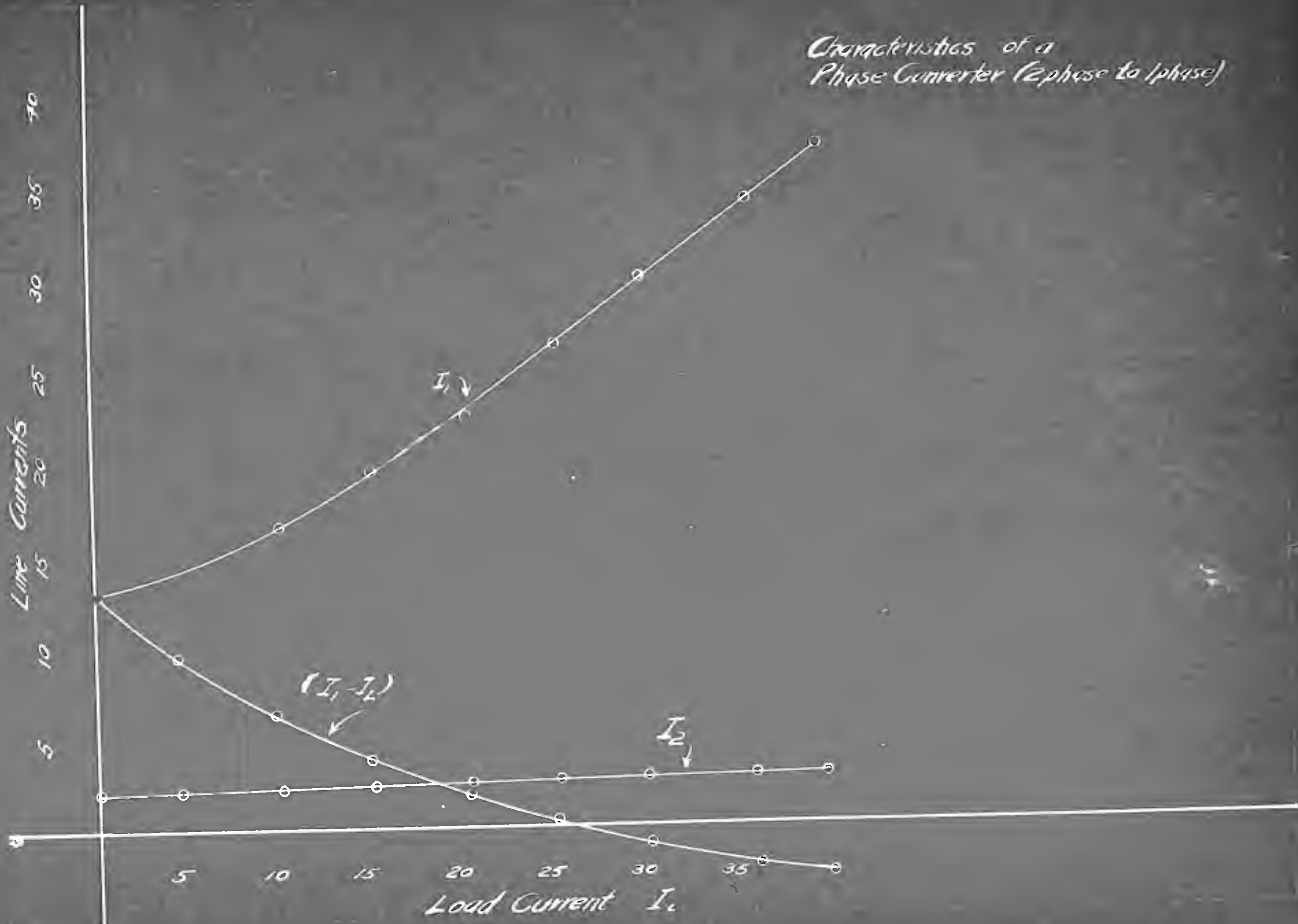
PART A.

A	V	W	A	V	W	W	A	Alt.
13.0	80.0	.20	2.4	79	130	200	0	50
14.3	78.5	.50	2.4	78	205	120	4.5	49.5
16.8	78.2	.85	2.4	78	290	30	10.01	49.5
19.6	77.9	1.15	2.5	78	375	55	15.1	49.5
22.3	77.0	1.45	2.5	78	445	120	20.4	49.5
26.4	76.2	1.75	2.6	78	535	205	25.3	49.5
30.0	76.0	2.05	2.7	78	600	265	30.0	49.5
34.0	75.0	2.49	2.8	77	680	340	36.0	49.5
37.0	74.5	2.60	2.9	77	740	395	39.6	49.5

PART B.

24.5	80	.40	00	85	400	405	0	50.
25.5	78	.85	1.0	83	495	395	5.2	50.
28.0	78	1.2	2.4	83.5	545	800	10.1	50.
30.8	77	1.6	3.5	82.0	760	780	15.	50.

Characteristics of a
Phase Converter (2phase to 1phase)



OPINION OF VARIOUS WRITERS.

Operating polyphase motors single-phase is a practice that in general, advised against, but under the conditions it would seem to have some advantages. This is shown by the following figures. A plant with two-phase generators supplying 125 K. W. lighting and power load with motors operating two-phase and single-phase, give the following figures:

Motors two-phase		Motors single-phase
Apparent power	= 47.4	37.125
True power	= 19.6	19.4
Power factor	= 19.6	19.4

It is seen then, that the other things being equal the power factor of the system is slightly improved, and the heating of the generators is reduced.

A three-phase motor can also be operated on a six circuit if the winding be disconnected and thus have six terminals from the machine.

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EXPERIMENT 2.

A STUDY OF A.C. INDUCTION INSTRUMENTS

The object in performing this experiment was to determine whether or not the accuracy of induction instruments is affected by a change in frequency and wave shape. This is important to the engineer as almost all Westinghouse switchboard meters are of the induction type.

The experiment was run on the following schedule.

Induction Ammeter:

Westinghouse make.

With a constant value of current, vary frequency from 60 cycles down to a low value, and compare readings of a G. E. ammeter with Westinghouse.

Induction Voltmeter:

Westinghouse made.

With a constant voltage, vary frequency from 60 cycles down to as low a value as practicable and compare readings of a

Weston electro-dynamometer type voltmeter with those of the Westinghouse.

Induction Wattmeter:

Westinghouse make.

With a constant power, change frequency from 60 cycles to about 30 cycles, applying half-voltage to pressure circuit of meter. Compare readings with those of a Weston electro-dynamometer type wattmeter.

If time is available test meters on a different wave form for any one current, voltage or power.

Westing-house Ammeter	G.E. Ammeter	Fre-quency.
Non Inductive Load		
8.9	9.0	60
8.92	9.0	55
8.95	9.0	50
8.98	9.0	45
8.99	9.0	40
9.00	9.0	35
8.975	9.0	30
8.900	9.0	25
Inductive Load		
8.9	9.0	60
8.95	9.0	55
8.95	8.95	50
8.92	8.90	45
8.88	8.85	40
8.80	8.80	35
8.66	8.70	30
8.46	8.55	25
8.10	8.30	20
7.50	7.80	15

Westing-house Voltmeter.	Weston volt-meter.	Fre-quency.
Volts		
75.0	80	60
74.1	80	55
73.5	80	50
72.3	80	45
71.0	80	40
68.6	80	35
64.3	80	30

Weston wattmeter	FRe-quency	Westing-house
NonInductive Load		
600	60	610
600	55	610
600	50	610
600	45	612.5
600	40	612.5
600	35	615.
600	30	620.
600	25	625.

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Westinghouse Ammeter	G.E. Ammeter	Fre- quency
Inductive Load - Corrected		
8.9	9.0	60
8.95	9.0	55
9.00	9.0	50
9.02	9.0	45
9.03	9.0	40
9.00	9.0	35
8.96	9.0	30
8.91	9.0	25
8.80	9.0	20
8.70	9.0	15

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INDUCTION AMMETER

Theory

The general explanation of the action of induction ammeters, voltmeters and wattmeters may be based on the fact that when two alternating fields are in time and space quadrature their resultant is a field which rotates in space. When a small aluminum cylinder, pivoted so that it is free to rotate, is placed in the field, eddy-currents are induced in it. The action of the field is to drag these currents with it and hence rotates the cylinder.

Calling the component fluxes of the rotating field ϕ_1 and ϕ_2, and γ the angle of lag of the induced currents I_1 and I_2 behind their respective induced voltages E_1 and E_2, Fig. B gives the time phase relations. Angles B_1 and B_2 are measured from an arbitrary line.

The turning moment is due to the reaction between ϕ_1 and I_2, less that between ϕ_2 and I_1. The flux waves are assumed to be sine waves.

The instantaneous torque is:

$$T = K i_1 \phi_2 - K i_2 \phi_1$$

average torque $T = \frac{1}{T}\int_0^T (K i_1 \phi_2 - K i_2 \phi_1)\, dT$

$$= K'\phi_2 I_1 \text{ Cos } (\gamma + 90^\circ + B_1 - B_2) - K\phi_1 I_2 \cos (\gamma + 90^\circ + B_2 - B_1).$$

$$= - K'\phi_2 I_1 \sin (\gamma - (B_2 - B_1)) + K'\phi_1 I_2 \sin (\gamma + (B_2 - B_1))$$

at a given frequency,

$$I_1 = \frac{K_1 \phi_1 f}{Z}$$

$$I_2 = \frac{K_2 \phi_2 f}{Z}$$

where Z is the impedence of the current paths in the rotor.

$$T = \frac{K'' f \phi_1 \phi_2}{Z} \left\{ \sin (\gamma - (B_2 - B_1) + \sin (\gamma + (B_2 - B_1)\right) = \frac{K''' f \phi_1 \phi_2}{Z} \cos \gamma \sin (B_2 - B_1) \quad \text{(A)}$$

The Westinghouse induction ammeter is shown in Fig. C. The line current enters at the terminals and flows through coil P, giving rise to flux ϕP, which crosses the air gap in a horizontal

WESTINGHOUSE INDUCTION TYPE AMMETER

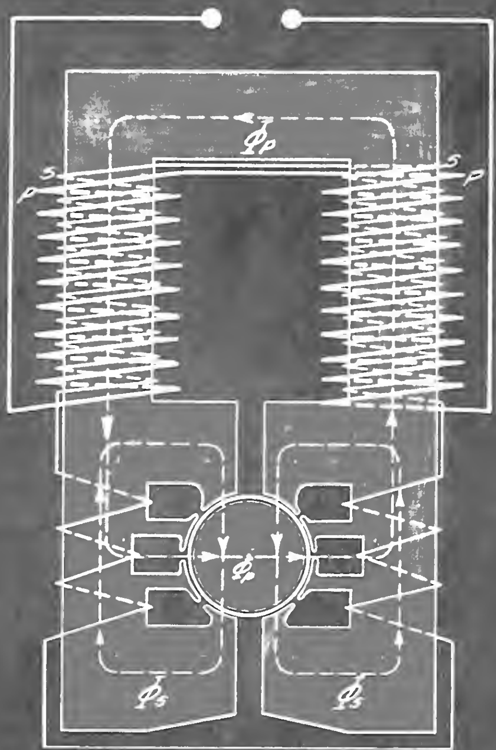


Fig. C

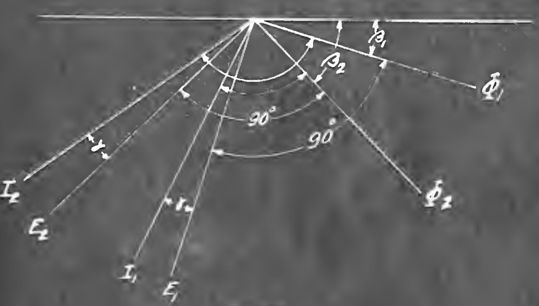


Fig. B

Fig. D

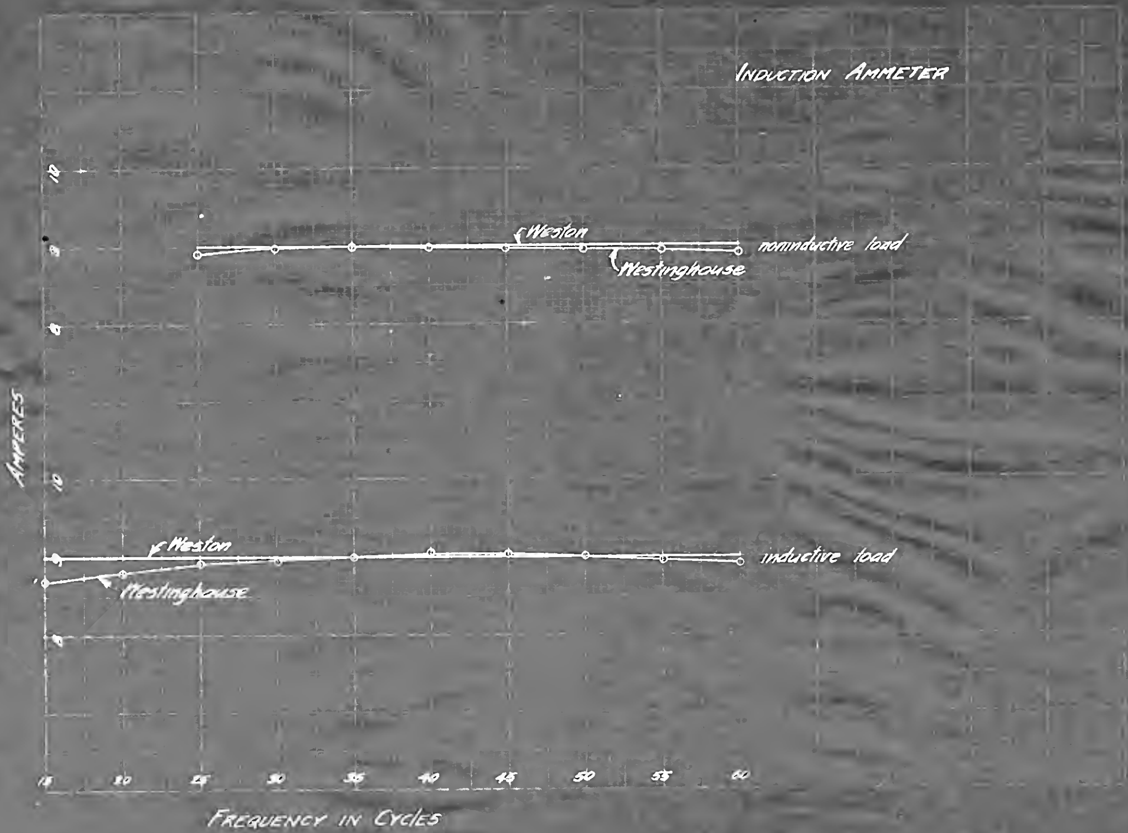
direction S; the secondary winding,with coils AA,forms a closed circuit ϕ_p induces a current in winding S which, in turn, produces ϕ_S which crosses the air gap in a vertical direction. The fluxes are in the proper space relation, and the proper time relations are obtained thru transformer action of the two windings.

Referring to the vector diagrams Fig. D, ϕ_p links both P and S;it is therefore the mutual flux, and induces in S an e.m.f., E_S, which lags 90° behind ϕ_p. The flux ϕ_S threads only S and not P,and is therefore the secondary leakage flux; it is in time phase with I_S. E_S is the vector sum of I_sX_s, the reactance drop due to ϕ_S,and I_sR_s, the ohmic drop in S. ϕ_p and ϕ_S are slightly more than 90° apart in time phase and will produce a torque on the rotor.

I is the line current so

$$\phi_S = K_2 I$$

$$\phi_p = \frac{K_3 I}{f}$$


INDUCTION AMMETER
Weston
nonınductive load
Westinghouse
Weston
inductive load
Westinghouse
AMPERES
FREQUENCY IN CYCLES

Substituting in equation (A)

$$T = K_4 I^2 \frac{\cos \gamma}{Z} \sin (B_2 - B_1)$$

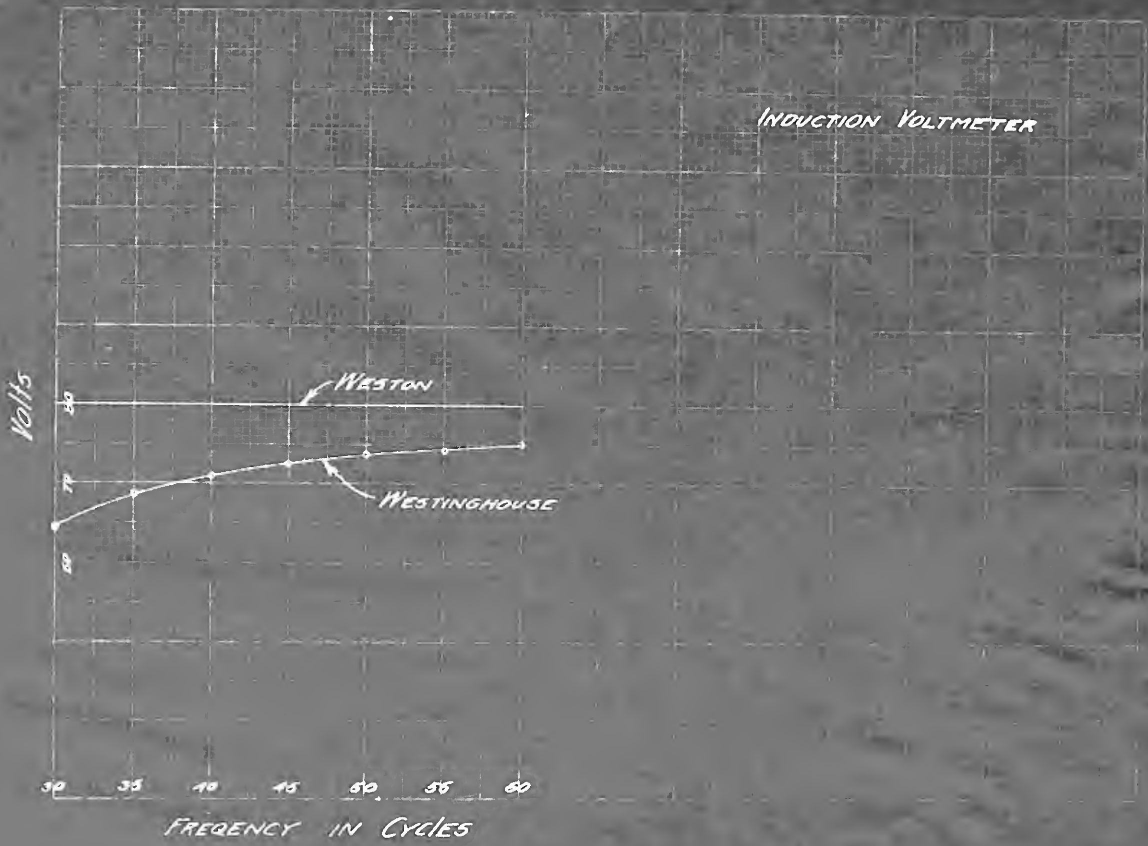
$$B_2 - B_1 = \text{approx. } 90^\circ.$$

Z, γ and $(B_2 - B_1)$ all vary with a change in frequency so it is necessary to design the instrument so the factor $\frac{\cos \gamma \sin (B_2 - B_1)}{Z}$ stays as constant as possible. The error between 25 cycles and 60 cycles may be reduced to as low as .5%.

Changes in temperature affect Z and γ for correct indication for any given current $\frac{\phi_S \phi_p}{2}$ cos γ must be made constant with respect to temperature. This is attained by giving the secondary a temperature coefficient $= \frac{Z}{\cos \gamma}$ by making it partly copper and partly of resistance wire of low temperature coefficient.

INDUCTION VOLTMETER

The induction voltmeter is the same as the ammeter except that the primary coil is of fine wire and is in series with an external non-inductive resistance of zero temperature coefficient. The ratio of ohmic resistance to total impedance is made very high so that the current in the primary coil is practically independent of frequency.


INDUCTION VOLTMETER
Volts
80
70
60
WESTON
WESTINGHOUSE
30
35
40
45
50
55
60
FREQENCY IN CYCLES

INDUCTION WATTMETER

The induction principle is used in switchboard wattmeters as well as in switchboard ammeters and voltmeters.

The core is made of finely laminated iron. The potential coil is wound around the legs of the core and the current through it magnetizes the core, as shown by the arrows in Fig. E. The potential coil flux, ϕ_1, passes through the air gap and rotor in a horizontal direction. The current coil gives rise to flux ϕ_2, which crosses the air gap and rotor in a vertical direction. The rotor is a thin aluminum cylinder pivoted so that it may rotate in the air gap.

The air gaps are made large so the fluxes will be proportional to their respective currents. The fluxes are in proper space relation for producing a torque on the rotor. The condition of correct time phase relation is attained as described below.

$(B_2 - B_1)$ is the time phase angle between the two

WESTINGHOUSE INDUCTION TYPE WATTMETER

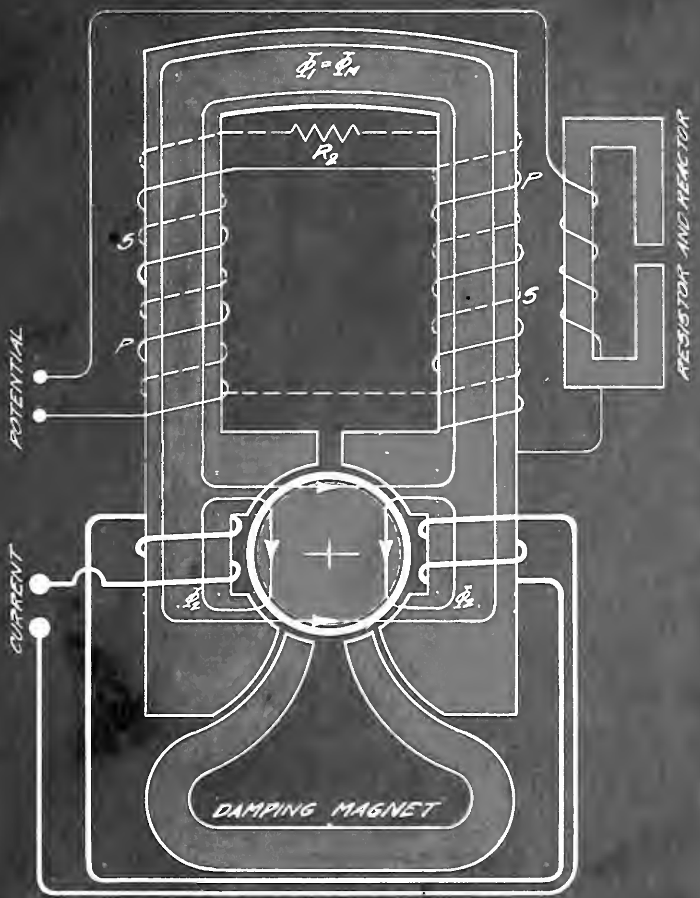


Fig. E

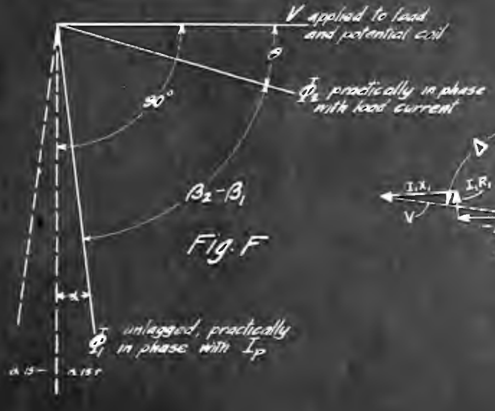


Fig. F

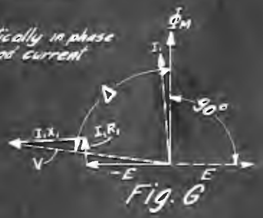


Fig. G

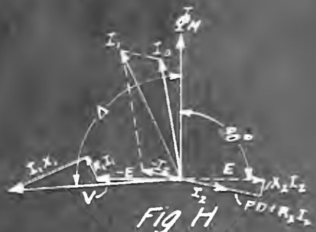


Fig. H

fluxes: The time phase diagram is Fig. F. The potential circuit is made highly inductive so ϕ_1 lags behind the applied voltage, V, by an angle of 75 or 80^o; it will not lag 90^o due to energy losses in the core and windings.

The angle α is the amount - by which ϕ_1 falls short of being in quadrature with V.

$$\alpha + (B_2 - B_1) + \theta = 90^o.$$

θ is the angle by which the line current, I, lags behind V. The torque, as in the ammeter, is at a fixed frequency,

$$T = K_5 VI \frac{\cos \gamma}{Z} \sin (B_2 - B_1)$$

$$= K_5 VI \frac{\cos \gamma}{Z} \sin (90 - \alpha - \theta)$$

$$= K_5 VI \frac{\cos \gamma}{Z} \cos (\alpha + \theta)$$

Power is VI cos θ, so if α is made zero, (ϕ_1 90^o behind V) the torque will be proportional to the power. This is, of course, the desired adjustment, and is attained by the use of a special phase shifting device which will increase the phase angle between the useful potential coil

flux in the air gap and the applied voltage.

The magnetic circuit of the potential coil should be such that the useful potential coil flux is naturally as near 90° from V as possible, i.e., α should be as near zero as possible; then the amount of additional lagging required will be less. The less the lagging to be caused by the phase shifting device the better the meter will work over wide changes in frequency. The phase shifting device in most common use in this country is a secondary winding, S, closed on itself through a resistance, R_2.

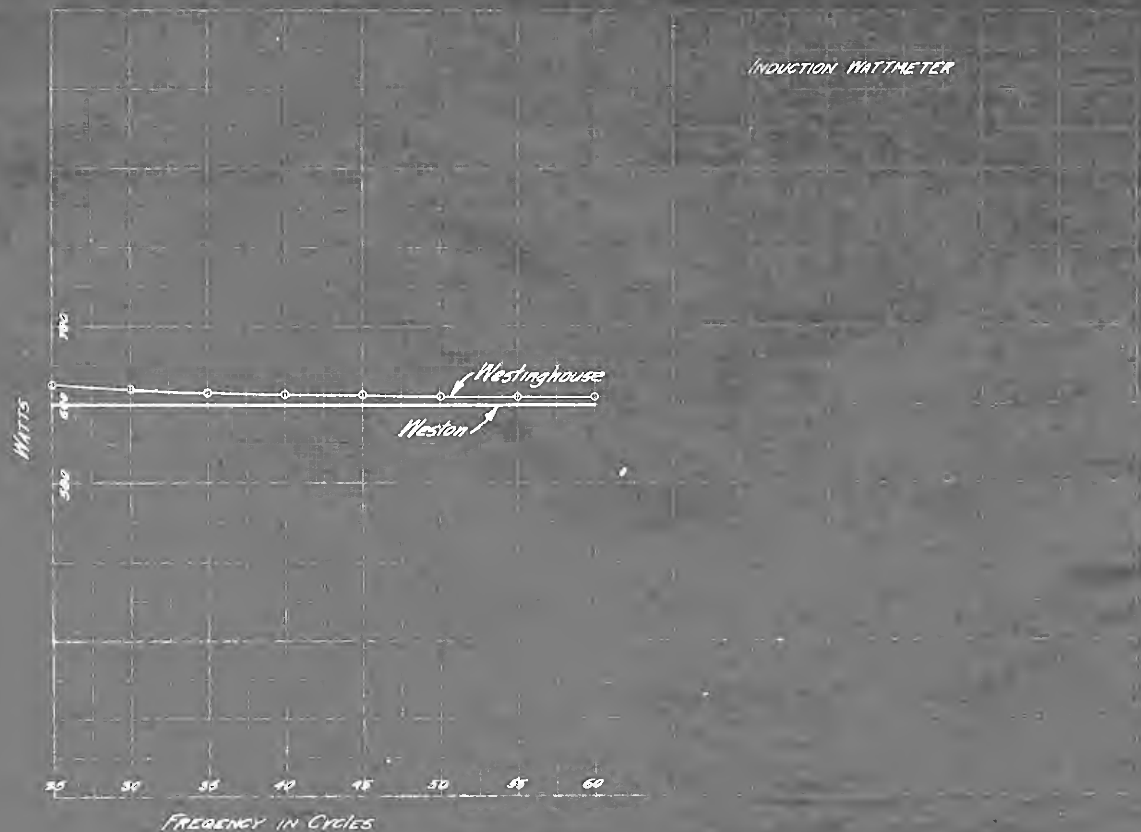
This lagging arrangement makes the potential circuit equivalent to a transformer with a large primary leakage flux, much of which is in the series reactor.

The primary winding of the potential circuit has a rather high resistance as well as a high reactance so that if the circuit through R_2 is open the instrument is under-lagged, that is, the angle between the useful potential coil

flux, ϕ_M, and V is less than 90^o.

There is assumed to be an equal number of turns in the secondary coil and the part of the potential coil wound on the main core (1 to 1 ratio). Referring to the vector diagram, Fig. G, ϕ_M, threads both the potential coil winding and the secondary and induces an e.m.f., - E, and, E, in each respectively. The applied voltage, V, must overcome the vector sum of - E, R_1I_1, (the primary ohmic drop) and X_1I_1 (the primary reactive drop, due to the primary leakage flux). As shown, the angle Δ between V and ϕ_M is less than 90^o.

A current, I_2, flows in the secondary circuit when it is closed through R_2. Fig. H is the vector diagram for this condition. I, is now the vector sum of - I_2 and I_o the magnetizing current. When the secondary is open I_1 = I_o. As may be seen in the diagram, I_1 is rotated counter-clockwise when R_2 is closed, thereby also rotating R_1I_1, X_1I_1, and V in the same



INDUCTION WATTMETER
Westinghouse
Weston
WATTS
25
30
35
40
45
50
55
60
FREQUENCY IN CYCLES

direction. This means that Δ is increased.

In order to adjust Δ to exactly 90° it is necessary to calibrate the instrument on a load having unity power factor and then on a load having a low power factor, say .5. If the results do not agree R_2 is varied until they do.

As a change in frequency changes the reactance and hence the value of Δ , an induction wattmeter will not indicate correctly for any frequency except that on which it was calibrated. As a change of frequency affects the indication, the wave form will also affect it.

Experiment No. 3

Polyphase Induction Watt-Hour Meter

Introduction

The watt-hour meter operates under more varied and exacting conditions than almost any other piece of apparatus. It is frequently subjected to vibration, moisture, short circuits, and extremes of temperature; it must measure accurately on varying voltages and various wave forms; it must operate for many months without any supervision or attention whatever; and, in spite of all these conditions it is expected to register with accuracy from a few per cent of its rated capacity to a fifty per cent overload. To become more familiar with this useful instrument and to understand and to know how to correct is errors when adjustment might be needed, was the object of performing the following experiment on the polyphase induction watt-hour meter.

Experiment No. 3

Polyphase Induction Watthour Meter.

The meter used in this test was a Westinghouse, Round Type, Polyphase induction watt hour meter. The method of precedure consisted in connecting the two current coils in series and the potential coils in parallel after disconnecting all the current transformer terminals from the instrument.

A wattmeter was used to measure the power and a stop watch registered the time. The time of fifty revolutions of the disk was recorded and from this notation the r.p.m. of the disk was calculated. From the formula $M = \frac{K\ R}{S}$, where M equals the watts passing through the meter, K, the meter constant (1200), R, the r.m.p. of the disk and S the duration of the test in seconds, the number of watts as registered by the meter under test may be obtained and compared to the standard wattmeter reading. The following data were taken at normal voltage, normal frequency and variation of load from zero to full load value of five amperes.

A non inductive load was used.

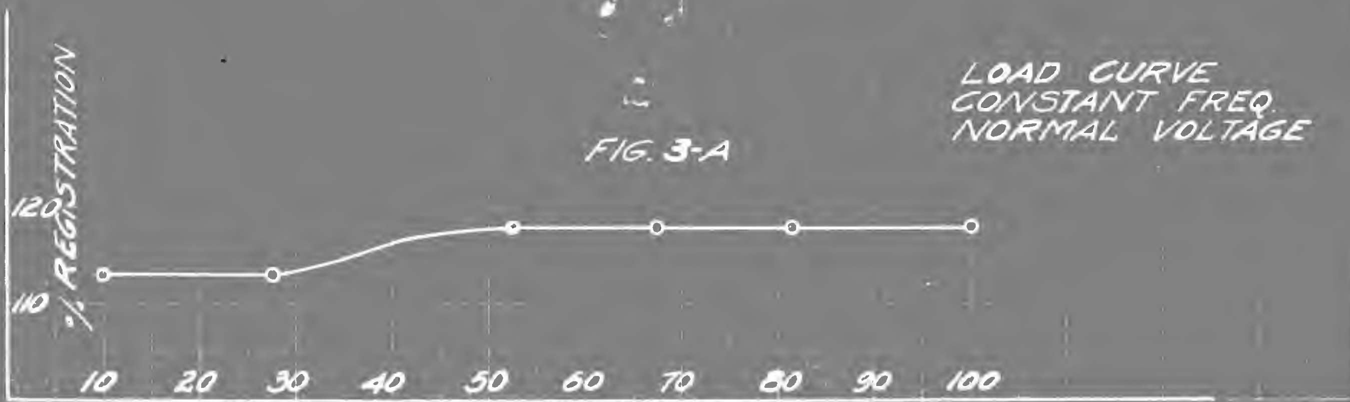
Load in Amp.	% Full Load	True Watts	Registered Watts	% Registration
0.485	9.7	92.	104.	113.
1.390	27.8	224.	252.	113.
2.620	52.5	482.	568.	118.
3.380	67.5	614.	728.	118.
4.080	81.5	740.	874.	118.
5.030	100.0	912.	1078.	118.

A curve of % registration plotted against % load as a results of the above data is shown below in figure 3-A.

Data for a frequency curve were then obtained at constant full load current and normal voltage. The results are noted below.

Cycles	True Watts	Registered Watts	% Registration
50.0	922.	1090	118
52.5	920.	1090	118
55.0	920.	1090	118
57.5	920.	1090	118
60.0	922.	1100	119


FREQUENCY CURVE
CONSTANT LOAD
NORMAL VOLTAGE
% REGISTRATION
120
110
FREQUENCY
50
52.5
55
57.5
60
FIG. 3-B


LOAD CURVE
CONSTANT FREQ.
NORMAL VOLTAGE
% REGISTRATION
120
110
10
20
30
40
50
60
70
80
90
100
FIG. 3-A

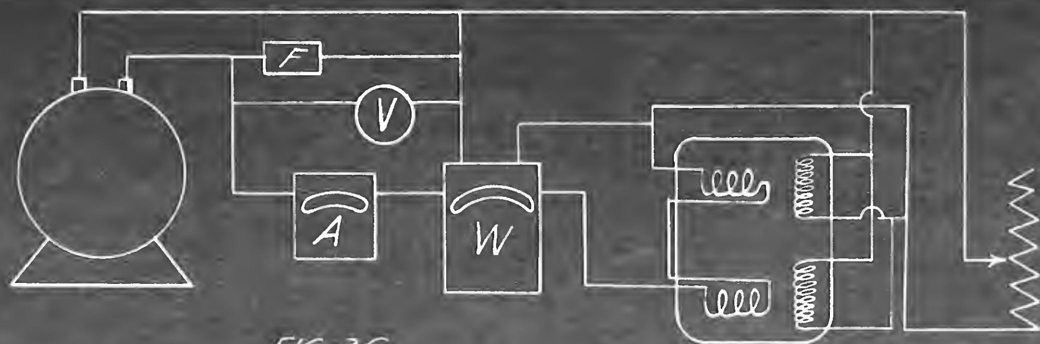
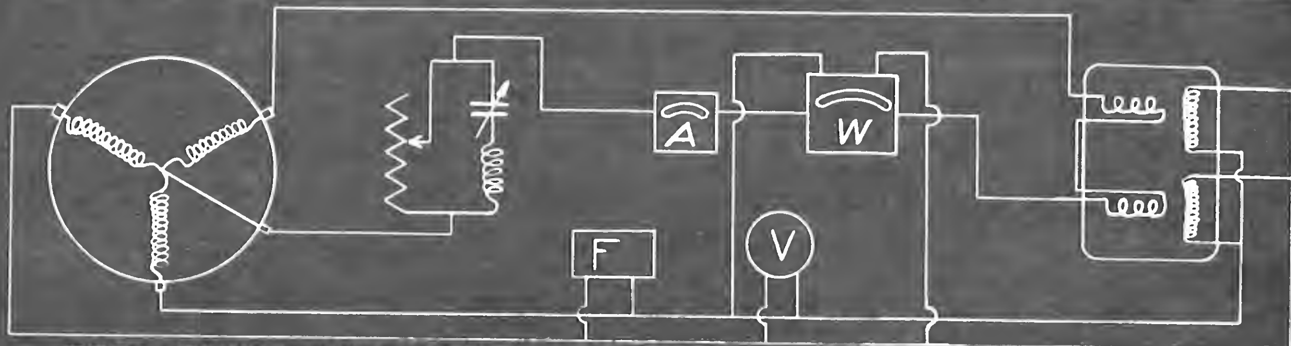


FIG. 3-C.

A curve of per cent registration and frequency is shown below illustrating no perceptible change within a range of 10 cycles. (fig. 3-B). The connections for carrying out the above work are sketched in fig. 3-C.

Part Two of Experiment 3.

The second part of this experiment consisted in obtaining zero power factor at normal full load current, normal voltage and frequency and observing the accuracy of the meter at this power factor. The scheme of connections is show in fig. 3-D. A 15 K.W. alternator is connected in star; the voltage terminals are taken off of two points of the Y while the current is tapped off from the remaining point to neutral. If the current coil and armature winding had no inductance the angle between the current and voltage would be exactly 90° but on account of the small inductive effect it was necessary to connect a condenser and inductance in series and the two in parallel with the load. The



ZERO POWER FACTOR METHOD FOR TESTING POLYPHASE WATTHOUR-METERS.

FIG. 3-D

FIG. 3-E

function of the inductance was to damp out the harmonics. A very fine adjustment was obtained in this way and a power factor of exactly zero was secured. In order to find out whether the current was leading or lagging an inductance was placed in the circuit. If the current was leading the voltage the wattmeter reading would decrease with the additional inductance. Since the current was found to be leading a variable condenser was placed in circuit so that exact quadrature might be obtained. When the voltage terminals were interchanged the current lagged 90° and a zero reading on the wattmeter was maintained. A vector diagram of the voltage and current in the system is shown in figure 3-E. The direction of the voltage is C B between two points of the Y and from neutral to the remaining point the voltage is N A and 90° to the first voltage. The current I_1 lags slightly behind the voltage N A, but by the addition of the capacity it may be adjusted to

be exactly ih phase with N A and in exact quadrature to C B, as shown by vector I. The disk of the meter rotates at zero power factor showing an improper adjustment of the power factor compensation; but from data taken the r.p.m. were found to be zero when the current was lagging the voltage by 90.1°. This completed the experimental work on this experiment and following is a description of the instrument and its mechanism in detail.

Discussion.

In measuring the power in a three phase three wire circuit the current coils of the two wattmeters are placed in any two of the three phase leads, and the pressure coils are connected, respectively, between these two leads and the third lead. A simple proof of the correctness of the two wattmeter method of measuring the power in a three phase circuit under any condition of service is given below. In fig. 3-F, let the sides of the triangle A B C represent the relative

values and phase positions of the three e.m.f.'s of an unsymmetrical three phase system. Assume the receiver to be delta connected, and let I_{AB} be the current in coil A B, and ϕAB be the angle of lag of this current with respect to the e.m.f. of the coil. Similarly, let I_{BC} and I_{AC} represent the value and phase position of the currents in coils BC and AC. No restriction is made as to the values of currents, e.m.f.'s or as to the several lag angles. Let one element of the polyphase watt hour meter be connected with its current coil in lead A, and its e.m.f. coil across this lead and lead C., also a wattmeter at B, with pressure coil between B and C. Each wattmeter will show a deflection, which may be represented by $W = E I \cos \phi$ where I is the amperes in the current coil, E is the volts across the e.m.f. coil, and ϕ is the angle between this e.m.f. and the current I. Assume, in the first place the current flowing in coil A B to be absent while measurements are made upon the watts supplied to the other coils, as indicated

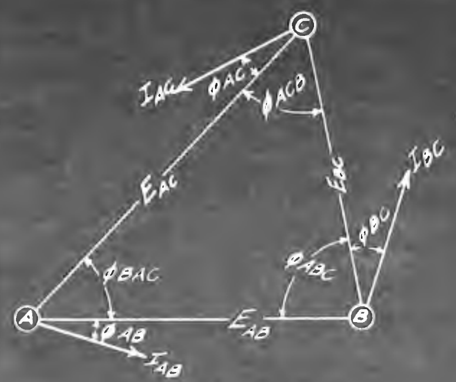


FIG. 3-F

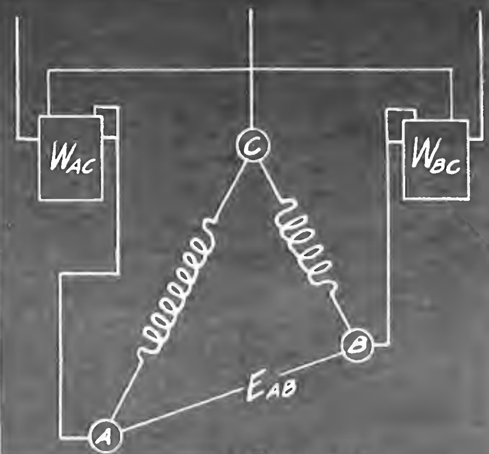


FIG. 3-G

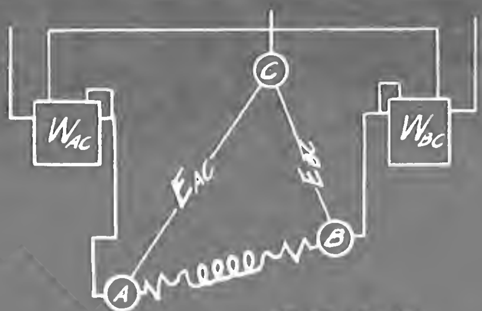


FIG. 3-H

in figure 3-G. Evidently the sum of the readings of the meters as connected gives the watts in the two remaining coils, since each meter is connected as though measuring power in a single phase circuit. Now assume, in the second place, the current to flow in coil A B alone while the currents in the other two coils are absent, as shown in fig. 3-H. The wattmeters as connected now register as their sum the true watts supplied to coil AB. When all three currents flow simultaneously, each wattmeter will show a deflection equal to the sum of its two previous readings, since its e.m.f. coil has undergone no change in connection and the two currents causing the former deflections are now superposed, and the true power transferred will be properly recorded by the two elements of the polyphase meter. Two wattmeters having their current coils in series with a given single phase load, and one terminal of the e.m.f. coil of each meter connected to the opposite

leads of the circuit supplying power to the load and the other two free terminals connected together and placed at any point of any relative potential compared with that of the load as shown in fig. 3-H, will give the true value of power transmitted.

The Westinghouse induction polyphase watthour meter is a combination of two single phase metering elements, the armature disks of which are mounted on the same shaft or spindle. Only one registering mechanism is thus necessary. The total driving torque is the sum of the torques exerted by the two actuating elements and the registration is proportional to the energy passing through both. The operating principle of the meter is that of an induction motor where the armature is the rotor and the rotating fields the stator. The main elements of this instrument are:

(1) The field producing element

(2) The moving element

(3) The retarding element

(4) The registering element

(5) The mounting frame and bearings

(6) the friction compensator

(7) the power factor adjustment

(8) Frequency adjustment

(9) the case and cover.

(1). The field producing element consists of the electro-magnetic circuit and the measuring coils. One of these coils connected in series with the circuit to be metered, is wound of few turns and is therefore of low inductance. The current through it is in phase with the current in the metered circuit. The other coil, connected across the circuit, is highly inductive, and therefore the current in it is nearly 90degrees out of phase with, and proportional to the voltage of the metered circuit across its terminals. Therefore when the current in the circuit is in phase with the voltage, the

currents in the meter coils are displaced almost 90° by means of the power factor adjustment. The coils are so mounted on the core that the currents in them produce a rotating or shifting field in the air gap, in somewhat the same manner that the currents in the primary windings of an induction motor produce a rotating field. The strength of the rotating field with 90° phase difference between the currents is proportional to the product of the currents in the two coils and therefore proportional to the product of current and voltage in the metered circuit. At any other powerfactor the field is proportional to this product multiplied by the sine of the angle of phase difference between the two meter currents. If the current in the voltage coil is in quadrature with the voltage of the metered circuit, at any power factor the sine of the angle of phase difference between the currents in the meter circuits will be equal to the cosins of

the angular displacement between the current and voltage in the meteréd circuit. Under these conditions therefore the strength of the shifting field is proportional also, to the power factor of the circuit. In other words, the strength of the rotating field is proportional to the product of the volts, amperes, and power factor and is therefore a measure of the actual power. Energy is consumed in the field producing element of the meter. It is upon the design of this element that the losses in the meter depend. Current is flowing through the shunt coil continuously even when no energy is being taken, and the higher the inductance of this coil the smaller will be the energy component of the constant flow. The series coil causes a loss of energy proportional to the square of the current flowing. It also causes a drop in voltage, both inductive and resistance, hence the resistance and inductance of the series coil of the meter should be as low as possible.

The magnetic circuit should be so designed that the increase of magnetic flux with high voltage or high current will not have retarding action but will act only to increase the torque. If the retarding effect be not prevented the meter will of course run slow at overloads.

The shifting nature of the field produced can be shown by reference to figures 3 I and of the magnetic flux produced by the two windings; the directions however are constantly reversing owing to the alternations of the current in the coils. Denoting the shunt and series pole tips by the letters shown in fig. 3-J gives a clear statement of the relation of the fields at each $\frac{1}{4}$ period. The signs plus and minus represent the instantaneous values of the poles indicated. Thus at one instant the pole tips A, C and A' of the potential coil are maximum plus, minus, and plus, respectively, because the instantaneous value of the current is maximum, while the

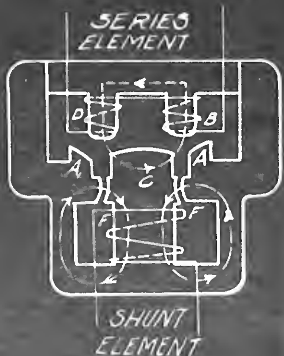


FIG 3-I

	A	B	C	D	A
START	+	0	–	0	+
$\frac{1}{4}$ PERIOD	0	–	0	+	0
$\frac{1}{2}$ PERIOD	–	0	+	0	–
$\frac{3}{4}$ PERIOD	0	+	0	–	0
FULL PERIOD	+	0	–	0	+

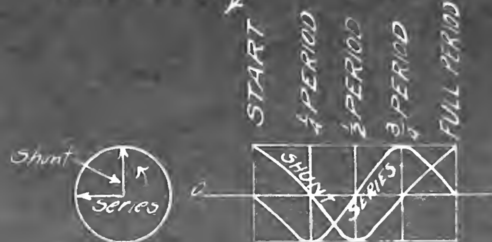


FIG. 3-J

value of the series flux is zero. At $\frac{1}{4}$ period later the current in the potential circuit is zero, giving zero magnetic potential at the pole tips, while the series current has reached a maximum value, giving maximum minus and plus at the pole tips B and D. At the next $\frac{1}{4}$ period the current in the potential circuit is again maximum, but in a direction opposite to what it was at the beginning, making the pole tips A, C and A' minus, plus and minus, respectively, while the series current again is zero. Continuing, the other relations of plus and minus poles shown in fig. 3-J is obtained. It will be observed from the table that both the plus and minus signs move constantly in the direction from A' to A, indicating a shifting of the field in this direction, the process being repeated during each cycle.

(2) The moving element consists of a light metal disk revolving through the air gap in which the rotating field is produced. The disk acts like the squirrel-cage armature of an

induction motor, developing the motive torque for the meter. This torque is counter balanced by the retarding element so that the speed is proportional to the torque. The disk should be made as light as possible to reduce the wear on the bearings to a minimum.

(3) The retarding element acts as a load on the induction motor and enables the adjustment of its speed to normal limits. In order that the speed shal be proportional to the driving torque, which varies with the watts in the circuit, it is necessary that the torque of the retarding device be proportional to the speed. For this reason a short circuited constant field generator consisting of a metal disk rotating between permanent magnet poles, has been generally adopted. The retading torque is produced by eddy currents which are induced in the disk in rotating through the magnetic field which opposes the force that produces them, thus developing a retarding torque. The constant field is produced for the retarding

disk by permanent magnets. The retarding disk may be the same disk used for the moving element in which case the meter field acts on one edge while the permanent magnet field acts on the edge diametrically opposite. This arrangement simplifies the number of parts and saves space and weight of moving element.

(4) The registering element mechanism comprises the dial pointers, and gear train necessary to secure the required reduction in speed. This gear train is driven directly by the rotor and therefore its friction should be low and constant.

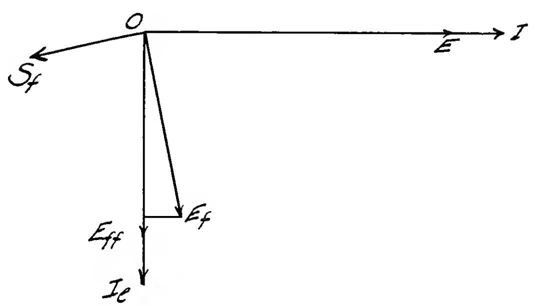
(5) The frame and bearings have an important fnfluence on the accuracy of the meter, as it is in the bearings that most of the friction in the meters occur. The frame should be rigid and free from vibration, so that the bearings will be at all times in perfect alignment. Initial friction is unavoidable in any meter construction and can be easily compensated for. A change in the

initial friction, however, due to wear of bearings, makes readjustment necessary.

(6) The friction in an induction-type meter is much less than in a commutator-type meter because of the absence of a commutator and an armature. On the other hand, the torque is less and the effect of friction at light load has to be compensated. The principle employed in practically all meters is that in which a flux is produced at the potential pole face, slightly out of phase with the main flux. Thus eddy currents will be produced in the disk which will be in phase with a small component of the main flux, giving rise to a slight torque which can be made sufficient to overcome the friction torque. This "out-of-phase" flux is produced in various ways in different meters. A common method is to place a short-circuited copper circuit or thin copper punching ("shading strip") in the potential-pole air-gap, in an unsymmetrical position, so that the desired unbalanced flux will be obtained. In the Columbia meter, the effect

(7) If the phase quadrature is not exact, the meter will obviously not register correctly under all conditions. In consequence of the ohmic resistance of the potential circuit, the current is never exactly 90° behind the impressed e.m.f. At any instant, the flux E_f from the potential pole, instead of being in phase with the eddy currents, I_e, due to the line current, is slightly behind as indicated. The torque is therefore proportional to the product I_e and oa, instead of I_e and E_{ff}. The meter will therefore run slow, but as a practical matter the error is so small that at unity power-factor it is insignificant. The error rapidly becomes large, however, as the power-factor decreases. As practically all alternating current circuits have a power-factor less than unity, a compensating coil is used to eliminate the error. This coil is a short-circuited coil placed on the potential pole and in which a current is induced 90° behind the generating (potential) flux. Its

flux, S_f will be in phase with that (induced) current and therefore 90° from E_f, with which it will combine. By adjusting the value of the resitance (lag adjustment) the resultant flux can be brought into exact phase with I_e and the meter will then register correctly on all power-factors. It is evident that with lagging power-factor in circuit, a meter will be slow if "under lagged" and fast if "over lagged." The opposite results will occur with a leading power-factor.



(8) The Frequency adjustment usually consists of a short circuited loop enclosing part or all of the shunt field flux and acts like the secondary of a transformer. The flux induces a current in it which acting with the current in the shunt coil, produces a slightly lagging field. By shifting the position of the resistance, of the short circuited loop, the lag may be so adjusted that the shunt field flux is in exact quadrature with the voltage. It should be noted that this adjustment makes the meter correct at or near one frequency only.

(9) The case and cover should be dust and bug proof, to avoid damage to the bearings, insulation and moving parts, and should be provided with means for sealing. Terminal chambers are so arranged that the cover of the meter need not be removed in connecting up. A window thru which the rotation of the disk should be provided for checking up the meter.

POLYPHASE WATT-HOUR METER CONNECTIONS

Obviously it is extremely important that the various circuits of a polyphase meter are properly connected. If, for example, the current-coil connections are interchanged and the line power-factor is 50 per cent, the meter will run at the normal 100 per cent power-factor speed, thus giving an error of 100 per cent.

A test for correct connections is as follows: If the line power-factor is over 50 per cent, rotation will always be forward when the potential or the current circuit of either element is disconnected, but in one case this speed will be less than in the other. If the power-factor is less than 50 per cent the rotation in one case will be backward.

When it is not known whether the power-factor is less or greater than 50 per cent, this may be determined by disconnecting one element and noting the speed. Then change the potential connection from the middle wire

to the other outside wire and again note the speed. If the power-factor is over 50 per cent the speed will be different in the two cases, but in the same direction. If the power-factor is less than 50 per cent, the rotation will be in opposite directions in the two cases.

USE OF INSTRUMENT TRANSFORMERS WITH WATT-HOUR METERS.

When the capacity of the circuit is over 200 amp. series-type instrument transformers are generally used to step-down the current to 5 amps. If the potential is over 440 volts, series transformers are almost invariably used, irrespective of the magnitude of current, in order to insulate the meter from the line; in such cases, shunt-type transformers are also used to reduce the voltage to 110 volts. The ratio and phase-angle errors of these transformers should be taken into account where high accuracy is important, as in the case of a large installation. These

errors can be largely compensated for by adjusting the meter speed. This completes the discussion of the various parts and concludes the experiment on the polyphase induction watt-hour meter.

Instruments

Westinghouse polyphase watt-hour Meter No. 9641.

Weston wattmeter, model 310, No. 375.

Weston voltmeter, No. 5091.

G. E. ammeter, (5-10) Type P-4, No. 202426.

Frequency meter, Hartmann - Kempf. No. 249195

Stop watch.

Carbon tube rheostat.

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EXPERIMENT NO. 4.

ANALYSIS OF WAVE FORMS.

OBJECT.

The consideration of wave forms is very important in the study of A. C. phenomena. It is useful because information can be obtained in the designing of electrical apparatus, or in the determination of the maximum e.m.f. and flux, and from that the determination of losses. The maximum stresses in the machine and from that the insulation to be used.

This, then is the scope of this experiment: given a certain machine and an oscillogram of its e.m.f. wave to analyze it into its component parts.

INTRODUCTION.

In any circuit one source of voltage wave distortion is in the alternator. With an open circuit the instantaneous voltage at the generator terminals is directly propertioned to the instantaneous cutting lines of force.

A simple conductor revolving in a uniform field at uniform velocity generates a sine voltage wave.

The distribution of flux however, may not be uniform, due to the slots and teeth in the armature which produce the pulsation of the field.

Other reasons of distortion are:

(1) Variation of speed of prime mover;

(2) Armature reaction; and

(3) The distribution of the armature conductors connected in series.

While the voltage wave of a single conductor has the same shape as the distribution of the magnetic flux at the armature circumfer-

ence, and so may differ considerably from a sine wave, the result of the waves of many conductors, and especially with a distributed armature winding, shows the higher harmonics in a much reduced degree, the resultant being nearer to a sine wave. In some cases the alternator can be designed so as to largely eliminate some of the higher harmonics, and thus produce a nearly pure sine wave. This, however, is not always desirable, or even if it is it cannot be accomplished easily.

For this reason, most of the alternators give more or less distorted waves. We say distorted, because of both theoretical discussions, and practical operation the sine wave is taken as standard, and any deviation is called distortion.

GENERAL SURVEY OF THE SUBJECT.

There are various ways of analysing wave forms obtained by curve tracers, or oscillographs the most important of which are:

(1) Mechanical: Harmonic Analyser

(2) Theoretical: Runge's method.

From these two methods the second one is to be taken up in detail.

Harmonic Analyser.

The harmonic analyser is simply a mechanical device used in connection with recording apparatus for obtaining quantitative values. The values of the sine and cosine coefficients are obtained separately for each one of the component waves, and from this coefficients, the resultant are found by calculation.

RUNGE'S METHOD.

Before attempting a final analysis of a wave perhaps it is desirable to note two fundamental characteristics.

[illegible]

[illegible]

[illegible]

[illegible]

(1) The waves are periodic, that is, the successive cycles are alike.

(2) The function is single valued.

According to Fourier (1822) any single valued, periodic function can be completely expressed by a simple trigonometric series, now known as Fourier's series.

$$y = K + A_1 \sin X + A_3 \sin 3X \text{------} A_n \sin nX$$
$$+ B_1 \cos X + B_2 \cos 2X + B_3 \cos 3X \text{--} B$$
$$B_n \cos nX \text{-----------------} (1)$$

For an e.m.f. wave the same series can be written as:

$$e = K + E_1 \sin (t\omega - \phi_1) + E_2 \sin (2\omega t + \phi_2) \text{------}$$
$$\text{----------} E_n \sin (n\omega t + \phi_n) \text{-------------------} (2)$$

where

K = constant $\quad e = y \quad X = t$

$E = \sqrt{A_1^2 + B_1^2}$ $\quad E_2 = A_2 + B_2$ etc.

$\phi_1 = \tan \frac{B_1}{A_1}$ $\quad \phi_2 = \tan$ etc.

In commercial power systems the A. C. voltage is produced by rotating machinery and hence the positive and negative halves of the waves are equal in magnitude and similar shape.

From this it follows that waves with equal positive and negative halves can have no even harmonics. Hence for voltage waves in power circuits,

$$e = E_1 \sin(\omega t + y_1) + E_3 \sin(3\omega t + y\ ) + E_5 \sin(5\ \ + y) \text{-----} E\ \ + \sin(2y - 1)\ \ t - y\ ) \text{-----} (3)$$

In the above equation the first term is called the fundamental, while the others are called 3rd, 5th etc. harmonics according to their frequency. The angular velocity for every turn is constant and can be found since

$$= 2\pi f$$

where

f = frequency.

The problem then is to find the coefficients E_1 E E_5 etc. also ------------ and from that write our final complete equation of instantaneous e. m. f.

EIGHTEEN POINT SCHEDULE.

Having traced our e.m.f. wave to be analysed, divide your base line of one-half cycle into eighteen parts; measure the ordinates and

ANALYSIS OF WAVE FORMS.

EIGHTEEN POINT SCHEDULE

MEAS. ORDIN.	SUMS	DIFFS	ANGLE	SINE TERMS 1ST AND 17TH	3D AND 15TH	5TH AND 13TH	7TH AND 11TH	9TH.
y_0		d_0						
y_1 y_{17}	S_1	d_1	sin 10°	s_1		$-s_7$	$-s_5$	
y_2 y_{16}	S_2	d_2	sin 20°	s_2		$-s_4$	$-s_8$	
y_3 y_{15}	S_3	d_3	sin 30°	s_3	b_1	s_3	$-s_3$	
y_4 y_{14}	S_4	d_4	sin 40°	s_4		s_8	s_2	
y_5 y_{13}	S_5	d_5	sin 50°	s_5		s_1	s_7	
y_6 y_{12}	S_6	d_6	sin 60°	s_6	b_2	$-s_6$	s_6	
y_7 y_{11}	S_7	d_7	sin 70°	s_7		$-s_5$	s_1	
y_8 y_{10}	S_8	d_8	sin 80°	s_8		s_2	$-s_4$	
y_9	S_9	d_9	sin 90°	s_9	b_3	s_9	$-s_9$	Σ

	1ST AND 17TH	3D AND 15TH	5TH AND 13TH	7TH AND 11TH	9TH.
	$A_1 = \frac{S'_o + S'_e}{9}$	$A_3 = \frac{S''_o + S''_e}{9}$	$A_5 = \frac{S'''_o + S'''_e}{9}$	$A_7 = \frac{S^{IV}_o + S^{IV}_e}{9}$	
	$S'_o = s_1 + s_3 + s_5 + s_7 + s_9$ $S'_e = s_2 + s_4 + s_6 + s_8$	$S''_o = b_1 + b_3$ $S''_e = b_2$	$S'''_o = -s_7 - s_5 + s_3 + s_1 + s_9$ $S'''_e = -s_4 + s_8 - s_6 + s_2$	$S^{IV}_o = -s_5 - s_3 + s_7 + s_1 - s_9$ $S^{IV}_e = -s_8 + s_2 + s_6 - s_4$	$A_9 = \frac{\Sigma}{9}$
	$A_{17} = \frac{S'_o - S'_e}{9}$	$A_{15} = \frac{S''_o - S''_e}{9}$	$A_{13} = \frac{S'''_o - S'''_e}{9}$	$A_{11} = \frac{S^{IV}_o - S^{IV}_e}{9}$	

$b_1 = s_1 + s_5 - s_7$

$b_2 = s_2 + s_4 - s_8$

$b_3 = s_3 - s_9$

$\Sigma = b_1 - b_3$

ANALYSIS OF WAVE FORMS.

EIGHTEEN POINT SCHEDULE.

	ANGLE	COSINE TERMS				
		1ST AND 17TH	3D AND 15TH	5TH AND 13TH	7TH AND 11TH	9TH
	sin 0°					
	sin 10°	d_8		$-d_2$	d_4	
	sin 20°	d_7		$-d_5$	d_1	
	sin 30°	d_6	δ_2	d_6	d_6	
	sin 40°	d_5		d_1	$-d_7$	
	sin 50°	d_4		d_8	$-d_2$	
	sin 60°	d_3	δ_1	$-d_3$	$-d_3$	
	sin 70°	d_2		$-d_4$	$-d_8$	
	sin 80°	d_1		d_7	d_5	
	sin 90°	d_0	δ_0	d_0	d_0	Δ
$\delta_0 = d_0 - d_6$ $\delta_1 = d_1 - d_5 - d_7$ $\delta_2 = d_2 - d_4 - d_8$ $\Delta = \delta_0 - \delta_2$		$B_1 = \frac{D_o' + D_e'}{9}$	$B_3 = \frac{D_o'' + D_e''}{9}$	$B_6 = \frac{D_o''' + D_e'''}{9}$	$B_{17} = \frac{D_o^{IV} + D_e^{IV}}{9}$	
		$D_o' = d_8 + d_6 + d_4 + d_2 + d_0$ $D_e' = d_7 + d_5 + d_3 + d_1$	$D_o'' = \delta_2 + \delta_0$ $D_e'' = \delta_1$	$D_o''' = -d_2 + d_6 + d_8 - d_4 + d_0$ $D_e''' = -d_5 + d_1 - d_3 + d_7$	$D_o^{IV} = d_4 + d_6 - d_2 - d_8 + d_0$ $D_e^{IV} = d_1 - d_7 - d_3 + d_5$	$B_9 = \frac{\Delta}{9}$
		$B_{17} = \frac{D_o' - D_e'}{9}$	$B_{15} = \frac{D_o'' - D_e''}{9}$	$A_{13} = \frac{D_o''' - D_e'''}{9}$	$A_{11} = \frac{D_o^{IV} - D_e^{IV}}{9}$	

write them down into two columns in the order indicated by y_0 y_1 in the eighteen point schedule.

In the next two columns appear the sums S of ordinates, found by adding those in the same row, and the differences D of the same ordinates.

In the fifth column are indicated the trigonometric functions which enter into the calculation. The rest of the schedule indicates in an abbreviated form what products are to be formed, the convention being adopted that each quantity S or D is to be multiplied by the sine of the angle which appears in the same row at the left. It is also to be noticed that each product which is involved in the calculation of any coefficient stands in the left, or right hand column, according as it depends upon odd, or even ordinates.

The arrangement and calculations according to eighteen point schedule are shown

		MEASURED ORDINATES		SU
0		0	0	[illegible]
1	.1736	.188	.200	.3
2	.3420	.288	.284	.5
3	.5000	.375	.430	.8
4	.6428	.508	.520	1.0
5	.7660	.585	.615	1.
6	.8660	.732	.790	1.
7	.9397	.800	.775	1.5
8	.9848	.785	.804	1.
9	1.000	.832		.8

$\delta_1 = .388 + 1.2 - 1.5$

$\delta_2 = .572 + 1.028-$

$\delta_3 = .805 - .832$

$\Sigma = .013 + 027$

SINE TERMS

		MEASURED ORDINATES		SUMS	DIFFS	1ST AND 17TH		3D AND 15TH		5TH AND 13TH		7TH AND 11TH		9TH
0		0	0	0	0									
1	.1736	.188	.200	.388	-.012	.06736				-.2735		-.2084		
2	.3420	.288	.284	.572	.004		.1962				-.3516		-.5435	
3	.5000	.375	.430	.805	-.055	.4025		.0065		.4025		-.4025		
4	.6428	.508	.520	1.028	-.012		.6608				1.0214		.3677	
5	.7660	.585	.615	1.200	-.030	.9192				.2972	-1.318	1.2085		
6	.8660	.732	.790	1.522	-.058		1.3181		.00953	-			1.3181	
7	.9397	.800	.775	1.572	.025	1.480				-1.127		.3646		
8	.9848	.785	.804	1.589	-.019		1.5698				.5633		-1.0124	
9	1.000	.832		.832	.832	.832		-.0270		.832		-.832		.04
						3.7011		.0205		.1306		.12836		
						3.7399		.0953		.085		.1289		
	$\delta_1 = .388 + 1.2 - 1.575 = .013$													
	$\delta_2 = .572 + 1.028 - 1.589 = .011$					$A_1 = .827$		$A_3 = .00122$		$A_5 = .00507$		$A_7 = .0287$		$A_9 = .00494$
	$\delta_9 = .805 - .832 = -.027$						$A_{17} = .0043$		$A_{15} = .00333$		$A_{13} = .0239$		$A_{11} = .000188$	
	$\Sigma = .013 + .027 = .04$													

		MEASURED ORDINATES		SU
0	0	0	0	[illegible]
1	.1736	.188	.200	.3
2	.3420	.288	.284	.5
3	.5000	.375	.430	.8
4	.6428	.508	.520	1.0
5	.7660	.585	.665	1.2
6	.8660	.732	.790	1.[illegible]
7	.9397	.800	.775	1.5
8	.9848	.785	.804	1.[illegible]
9	1.000	.832		.8

$\delta_0 = 0 + .058 = .$

$\delta_1 = -.012 + .030 -$

$\delta_2 = .004 + .012 + .0$

$\Delta = .058 - .035 =$

ANALYSIS OF WAVE FORMS

COSINE TERMS

		MEASURED ORDINATES		SUMS	DIFFS	1ST AND 17TH		3D AND 15TH		5TH AND 13TH		7TH AND 11TH		9TH
0	0	0	0	0	0									
1	.1736	.188	.200	.388	-.012	-.00829				-.000694		-.00208		
2	.3420	.288	.284	.572	.004		.00855				.01026		-.004104	
3	.5000	.375	.430	.805	-.055	-.029		.0175		-.028		+.029		
4	.6428	.508	.520	1.028	-.012		-.01928				-.00771		-.01607	
5	.7660	.585	.665	1.200	-.030	-.00919				-.01455		-.00306		
6	.8660	.732	.790	1.522	-.058		-.04763		-.00606		.04763		.04763	
7	.9397	.800	.775	1.572	.025	.00375				.011276		.01785		
8	.9848	.785	.804	1.589	-.019		-.011817				.02462		-.0295	
9	1.000	.832		.832	.832	0		.058						.023
						-.02773		.0355		-.03297		-.01629		
						-.07018		-.00606		.07479		-.002088		
	$\delta_0 = 0 + .058 = .058$					$B_1 = .019873$		$B_3 = .00771$		$B_5 = .00464$		$B_7 = .0024$		$B_9 = .00258$
	$\delta_1 = -.012 + .030 - .025 = -.007$						$B_{17} = .004716$		$B_{15} = .00306$		$B_{13} = -.01197$		$B_{11} = -.00178$	
	$\delta_2 = .004 + .012 + .019 = .035$													
	$\Delta = .058 - .035 = .023$													

in the next two pages, where the resultant coefficients and final e.m.f. equation appear.

CHECK ON THE EIGHTEEN POINT ANALYSIS.

In work of this kind some check on the accuracy of the numerical work is almost indispensable. Fortunately, such a check may be made without any considerable amount of labor.

The equations below give sufficient relations between the coefficients and the measured ordinates to establish the correctness of the values of the coefficients derived by calculations:

$$y_3 = (B_1 + B_{17}) + (B_3 + B_{15}) + (B + B) + (B + B) + B_9$$

$$y_9 = (A_1 + A_{17}) + (A_5 + A_{13}) + A_9 - (A_3 + A) - (A_7 + A_{11})$$

$$= 2 \sin 60 \quad (B_1 - B_{17}) - (B_5 - B_1) - (B - B_{11})$$

$$= 2 \sin 60 \quad (A_1 - A_7) - (A_5 - A) + (A - A_{11})$$

The above check applied to our analysis shows satisfactory results and adds confidence to the final outcome of the work.

$$C_1 = \sqrt{.827^2 + .01086^2} = .828 \qquad \theta_1 = 179^\circ\ 15'$$

$$C_3 = \sqrt{.00122^2 + .007715^2} = .00782 \qquad \theta_3 = 9^\circ\ 0'$$

$$C_5 = \sqrt{.0051^2 + .004647^2} = .00686 \qquad \theta_5 = 47^\circ\ 36'$$

$$C_7 = \sqrt{.0287^2 + .002042^2} = .0294 \qquad \theta_7 = 176^\circ\ 2'$$

$$C_9 = \sqrt{.00444^2 + .00255^2} = .00512 \qquad \theta_9 = 60^\circ\ 8'$$

$$C_{11} = \sqrt{.000188^2 + .01787^2} = .0179 \qquad \theta_{11} = 6^\circ\ 0'$$

$$C_{13} = \sqrt{.0239^2 + .01137^2} = .0263 \qquad \theta_{13} = 153^\circ\ 23'$$

$$C_{15} = \sqrt{.00333^2 + .00905^2} = .00965 \qquad \theta_{15} = 20^\circ\ 12'$$

$$C_{17} = \sqrt{.0043^2 + .004716^2} = .00638 \qquad \theta_{17} = 42^\circ\ 26'$$

FINAL E.M.F. EQUATION.

$$\begin{aligned} e = {} & .28 \sin(wt + 45') + .00782 \sin 3(wt + 3^\circ) \\ & + .00688 \sin 5(wt + 9^\circ 31') + .0294 \sin 7(wt + 25^\circ 9') \\ & + .00512 \sin 9(wt + 6^\circ 41') + .0179 \sin 11(wt + 33') \\ & + .0263 \sin 13(wt + 11^\circ 48') + .00965 \sin 15(wt + 1^\circ 21') \\ & + .00638 \sin 17(wt + 2^\circ 29'). \end{aligned}$$

OR

$$\begin{aligned} e = {} & 100 \sin(wt + .75^\circ) + 2.73 \sin 3(wt + 3.0^\circ) \\ & + 2.46 \sin 5(wt + 9.5^\circ) + 10.5 \sin 7(wt + 25.15^\circ). \\ & + 1.83 \sin 9(wt + 6.66^\circ) + 6.39 \sin 11(wt + .55^\circ) \\ & + 9.41 \sin 13(wt + 11.8^\circ) + 3.45 \sin 15(wt + 1.35) \\ & + 2.28 \sin 17(wt + 2.47). \ldots\ldots\ldots\ldots (4) \end{aligned}$$

SUMMARY.

Thus we have a method by which any wave obtained by a curve tracer, or oscillograph can be analysed into its component parts. The various coefficients, amplitudes and phases are calculated, and the final equation is written in a very satisfactory manner. Runge's method has an arrangement and check such that it gives very satisfactory results and facilitates the location of numerical errors.

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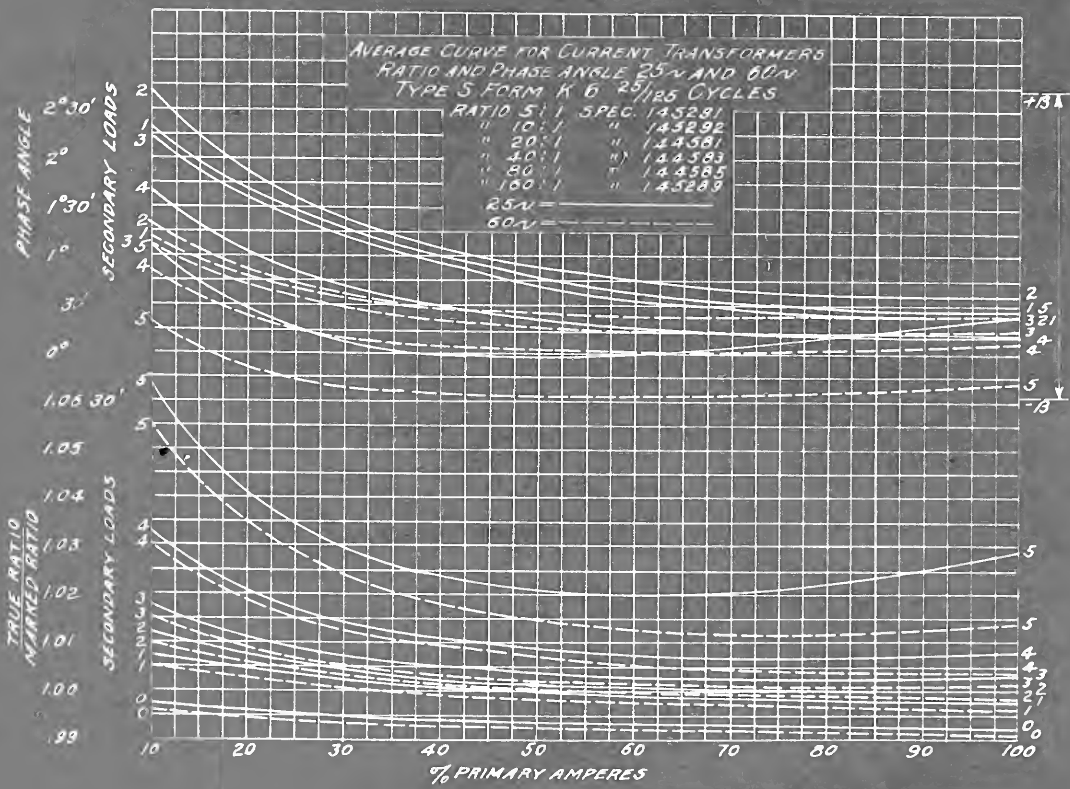
FIELD TESTING OF INSTRUMENT TRANSFORMERS BY AGNEW'S METHOD

Introduction

In current transformers the ratio of transformation and the phase angle between the actual vector position of the secondary current and the primary current reversed are not constants under various conditions of loading. The curves on the following page show the characteristics of a current transformer tested by the General Electric Co. From that curve sheet it is clear that when using a current transformer for accurate measurements the characteristics must be known. Various methods for determining such characteristics have been devised.

P. G. Agnew of the Bureau of Standards described a method of comparing instrument transformers with a standard of known characteristics in the November 21st issue of the Electrical World. This method proved to be very valuable for field testing and for small lab-


AVERAGE CURVE FOR CURRENT TRANSFORMERS
RATIO AND PHASE ANGLE 25~ AND 60~
TYPE S FORM K 6 25/125 CYCLES
RATIO 5:1 SPEC. 145281
" 10:1 " 145292
" 20:1 " 144581
" 40:1 " 144583
" 80:1 " 144585
" 160:1 " 145289
25~
60~
PHASE ANGLE
SECONDARY LOADS
TRUE RATIO / MARKED RATIO
% PRIMARY AMPERES
2°30'
2°
1°30'
1°
30'
0°
1.06 30'
1.05
1.04
1.03
1.02
1.01
1.00
.99
10
20
30
40
50
60
70
80
90
100
+β
-β

oratory work. H. M. Crothers gives simplified calculations for the method in the March 15, 1919 issue of the Electrical World. His method of procedure was followed throughout this experiment.

DETAILS OF THE METHOD

The essential apparatus used in this experiment consists of a standard and the test transformer with their primary windings connected in series with the supply circuit as shown in Fig. 1. The secondaries are connected to the current coils of two portable standard wattmeters, and a four-pole switch is interposed for the purpose of switching the meters from one transformer to the other. The voltage coils are connected either in series or in parallel according to the voltage available and are controlled by a common snap switch. If the source of supply is such that the desired variations in current are unattainable an auxiliary transformer must be used.

CHARACTERISTICS OF THE STANDARD USED

Westinghouse transformer #179 50/5

Secondary Current	Phase Angle	Ratio
0.91	52.5'	10.25
1.49	29.2'	10.19
1.90	19.2'	10.16
2.59	8.0'	10.14
2.88	4.3'	10.13
3.48	-0.9'	10.12
3.92	-4.3'	10.11
4.42	-7.5	10.10
4.95	-9.6	10.09

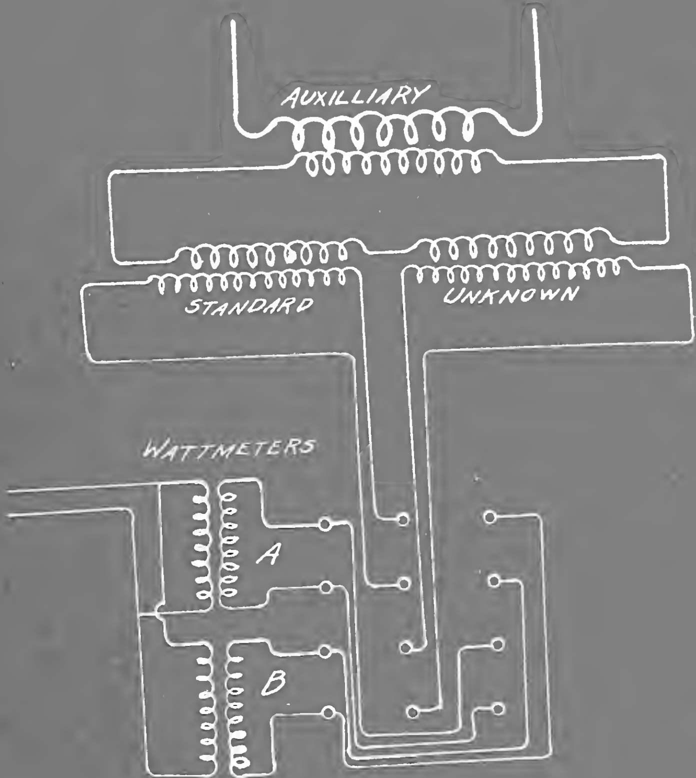
THEORY OF THE METHOD

H. M. Crothers gives the theory as follows:

With a chosen primary current flowing the ratio between the readings of the meters, will depend upon their adjustments and upon the current ratios of the transformers through which they are supplied, or

$$A_s/B_x = (R_xK_a)/(R_sK_b) \qquad (1)$$

in which R_s = the current ratio of the standard



AUXILLIARY
STANDARD
UNKNOWN
WATTMETERS
A
B

transformer; R_x = the current ratio of the test transformer; K_a = a constant depending upon the adjustment of meter A; K_b = a constant depending upon the adjustment of meter B; A_s = registration of meter A connected through the standard transformer, and B_x = registration of meter B connected through the test transformer.

If the meters be interchanged in the transformer secondaries and another pair of readings taken with the same current, then

$$A_s/B_x = (R_s K_a)/R_x K_b \qquad (2)$$

or dividing the equations member by member eliminates the meter constants and gives

$$(A_s B_s)/(A_x B_x) = (R_x/R_s)^2,$$

$$\text{or } R_x/R_s = \sqrt{(A_s B_s)/(A_x B_x} \qquad (3)$$

Agnew shows that this equation may be put in the form

$$(R_x - R_s)/R_s = \tfrac{1}{2} \quad (A_s - A_x)/A_x$$

$$(B_s - B_x)/B_x \qquad (4)$$

To determine the difference between phase angles of the two transformers it is necessary

to operate the meters at a low power factor. The registrations, then, are affected both by the current ratio and by the phase shifts of the secondary currents. If the voltage be shifted by the angle θ (measured positive when the voltage leads the current) and another set of readings (A' s, B' x, A' x, B' s) is taken, then the angle $_x$, by which the secondary current leads the reversed primary current according to Agnew is

$$\delta_x \text{ (in minutes)} = \delta_s - 3438/\tan\theta \ \ (A'_s - A'_x)/2\,A'_x \ \ (B'_s - B'_x)/2\,B'_x - (R_x - R_s)/R_s)$$

This is equivalent to the form given below in the discussion.

SIMPLIFIED CALCULATIONS

Use of this method in testing a large number of transformers has led to the development of simpler forms of the equations given. For the calculation of the current ratio of the test transformer a form is used which may be obtained from equation (3) by use of two simple expe-

dients.

The first and most obvious steps is, when taking the readings, to make $A_s = A_x$ and equal to some integral number of revolutions. Doing this, equation 3 becomes

$$R_x/R_s = \sqrt{B_s/B_x} = \sqrt{1 + (B_s - B_x)/B_x}$$

Expanding by the binomial theorem, all terms after the second may be dropped with a resulting error of the order of 0.01 per cent. This gives

$$R_x/R_s = 1 \quad \tfrac{1}{2} + [(B_s - B_x)/B_x] \quad \text{or}$$

$$(R_s - R_x)/R_s = \tfrac{1}{2} \ [(B_s - B_x)/B_x] \qquad (5)$$

The same form is obtained immediately from equation (4) when $A_x = A_s$.

The second expedient is the use of the "ratio factors" F_s and F_x instead of the ratios. As used by the Bureau of Standards, this term is defined as the true ratio divided by the marked ratio. It is convenient to state the ratio factor in per cent

F_s (in per cent) $= 100\ R_s/R_m$ (marked ratio).

The use of ratio factor is found helpful not only in this case but in all work with instrument transformers. Meters connected through instrument transformers usually have their scales or dials numbered to take account of the marked ratio of transformations so that if the ratio factor is stated in per cent the percentage error in the readings due to transformers is at once apparent.

The ratio factors are obtained by multiplying the numerator and denominator of the left member of equation (5) by $100/R_m$, giving

$$(F_x - F_s)/F_s = \tfrac{1}{2}\left[(B_s - B_x)/B_x\right],$$

$$\text{or } F_x = F_s + F_s\left[(B_s - B_x)/2\,B_x\right] \qquad (6)$$

The second term of the right member of equation (6) is of the order of 1 per cent of the first term. Therefore a few approximations in the second term will not affect appreciably the final result of the calculations. For F_s the value 100 may be substituted since the ratio factor of the standard transformer is

always very close to 100 per cent. Again for B_x the value A_s, which is very nearly equal to it, may be substituted. A_s is an even number and makes the arithmetic easier. Finally,

$$F_x = F_s + 100 \left[(B_s - B_x)/2A_s \right] \qquad (7)$$

(F_s and F_x are stated in per cent.)

$$(A_s = A_x).$$

The error in the value obtained for F_x due to all approximations involved will not exceed 0.1 per cent when the difference between the two transformer ratios is not over 3 per cent and the difference between the meter rates is under the same limit.

To calculate the phase angle δ_x the following line of reasoning may be used: It may be seen from equation (7) that $100 \left[(B_s - B_x)/2A_s \right]$ gives the percentage difference in the ratio factors of the transformers. The only new element affecting the second set of readings taken at low power factor is the fact that the secondary currents are displaced by different

angles from the primary current, and therefore the power factor at which a meter operates depends on the transformer through which it is connected. The difference then between $100[(B'_s - B'_x)/2A'_s]$ and $100[(B_s - B_x)/2A_s]$ will give the percentage difference in the power factors $\cos(\theta - \delta_x)$ and $\cos(\theta - \delta_s)$.

or $100[\cos(\theta - \delta_s) - \cos(\theta - \delta_x)]/[\cos(\theta - \delta_s)] = \pm 100[(B'_s - B'_x)/2A'_s - (B_s - B_x)/2A_s]$ (8)

The left member reduces approximately to $(\delta_x - \delta_s)\tan\theta/34.38$, so

$$\delta_x = \delta_s \pm (34.28/\tan\theta)[100(B'_s - B'_x)/2A'_s - 100(B_s - B_x)/2A_s]$$

When a three-phase supply is used, θ is usually 60 deg.

$$\text{so } \delta_x = \delta_s \pm 20[100(B'_s - B'_x)/2A'_s - 100(B_s - B_x)/2A_s] \quad (\theta = 60^\circ) \qquad (10)$$

The sign of the bracketed expression is determined by the definition of and θ and by the kind of transformers under test. With defined as the

small angle by which the secondary current (or voltage) leads the reversed primary current (or voltage) and θ defined as the angle by which the supplied voltage leads the supplied current, equation (9) becomes for current, transformers,

$$\alpha_x = \alpha_s - (34.38/\tan\theta)\left[100\,(B'_s - B'_x)/2A'_s - 100\,(B_s - B_x)/2A_s\right] \tag{11}$$

For voltage transformers $\alpha_x = \alpha_s + (34.38/\tan\theta)\left[100\,(B'_s - B'_x)/2A'_s - 100(B_s - B_x)/2A_s\right]$ (12)

TO DETERMINE WHETHER θ IS POSITIVE OR NEGATIVE

When $A'_s = A'_x$ equation (11) shows that $(B'_s - B'_x)$ depends only on the characteristics of the transformer and is independent of the characteristics of the meters. Likewise it can be shown that $(B'_s \;\; B'_x)$ depends only on the characteristics of the meters and is independent of the characteristics of the transformers. These relations can be utilized as follows: In the laboratory, or in some installation where the order of phase rotation is known, let a set of readings be taken at some chosen current through the

meters and with $\theta = 60$ deg. Suppose $B'_s + B'_x = N'$. Take another set of readings with the same values for the A readings and with $\theta = -60$ deg. If there is a small difference in the phasing adjustments of the two meters, the sum of the B'' readings will be different from N', say $B''_s + B''_x = N''$. On the subsequent tests at other places shift the voltage taps at random and take the readings for the phase-angle determination. At the current previously chosen again make the A readings the same as used before, and if the sum of the B readings is equal to N', the angle is positive. If the sum is equal to N", the angle is negative.

EFFECT OF INDUCTANCE IN THE CURRENT CIRCUITS

In the preceding discussion it has been assumed that the currents in the transformers and meters are in phase with the voltage to which they are connected. Experience shows that this is sometimes far from being the case. Instances have been met in which the current lagged

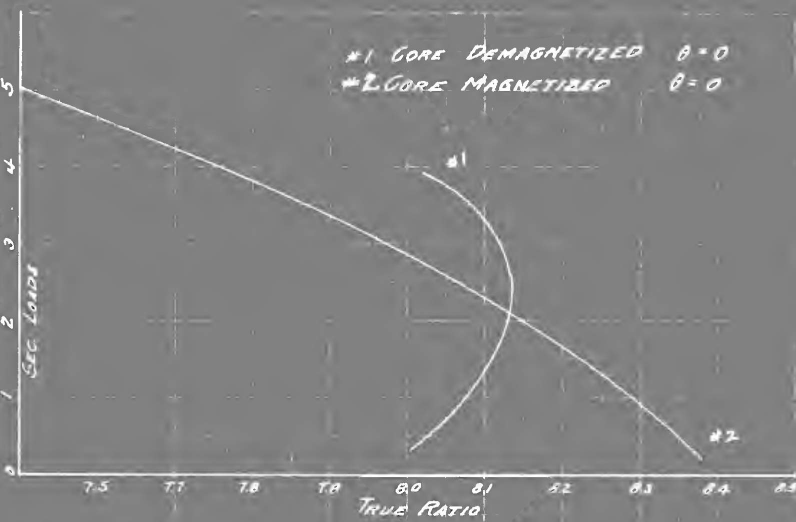
30 deg. or more. It is necessary to be always on guard against this possibility, for neglect to do so may lead to large errors.

It will be recalled that in determining the ratio of the transformer it was assumed that the meters were operating at a high power factor so that the effect of the transformer phase angles would be negligible. However, the effects of the phase angles are not negligible, generally, if the meters are operating at a power factor of 0.96 or lower. This corresponds to a lag of the current behind the voltage of 15 deg. or more. If the angle is 15 deg., equation (7) may be applied with a possible error due to this cause as high as 0.2 per cent; it will be less than 0.1 per cent if the phase angles of the two transformers differ by 30 min. or less.

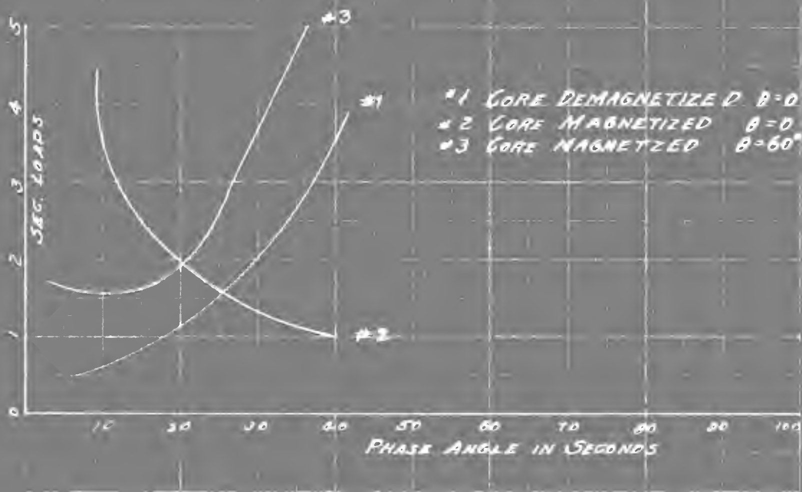
In the phase-angle determination the possible error comes from assuming that the angle θ between current and voltage on the meters is 60 deg. when it actually may be more or less by 15 deg. If the true angle is known, no error need

arise. A power-factor meter may be used, of course, if one is available. Another method used in the field with fair success involves timing the meters first with the voltage taps across the same phase as the current leads, then with the taps across the adjacent phase. If the watt-hour meters are correctly phased, the power factor may be determined within a few per cent from the ratio of the readings.

It is best to keep the current nearly in phase with the supply voltage by keeping the resistance of the current circuit as high as possible. A service voltage of 220 is better, therefore, than 110 volts, but 440 volts is still better if suitable rheostats and watt-hour meters are available. It has always been found possible to keep the phase angle between the current and voltage within the 15-deg. limit of 60 deg. by keeping the resistance of the control rheostat up to at least 20 ohms.



#1 CORE DEMAGNETIZED θ=0
#2 CORE MAGNETIZED θ=0
#1
#2
SEC. LOADS
0
1
2
3
4
5
7.5
7.7
7.8
7.9
8.0
8.1
8.2
8.3
8.4
8.5
TRUE RATIO



#1 CORE DEMAGNETIZED θ=0
#2 CORE MAGNETIZED θ=0
#3 CORE MAGNETZED θ=60°
#3
#1
#2
SEC. LOADS
0
1
2
3
4
5
10
20
30
40
50
60
70
80
90
100
PHASE ANGLE IN SECONDS

OBSERVATIONS

Cores demagnetized $\theta = 0$.

A_x	A_s	I_s	B_s	B_x	I_x
252	160	3.4	190	192	3.9
170	110	2.08	130	130	2.58
90	52	1.00	68	68	1.25
68	42	0.40	50	50	0.80

Core magnetized $\theta = 0$.

45	050	.50	30	50	.45
105	100	1.1	80	140	1.1
150	150	1.70	120	190	1.7
175	170	1.90	140	200	1.8
205	200	2.5	165	250	2.5
230	220	3.1	180	280	3.03
255	250	3.60	200	300	3.57
270	260	4.1	210	320	4.0
300	280	4.62	235	350	4.60
315	300	5.00	250	380	5.00

Core Magnetized $\theta = 60^{\circ}$

A_x	A_s	I_s	B_s	B_x	I_x
85	85	1.1	60	100	1.0
160	160	2.6	125	200	2.05
235	235	2.8	170	290	2.75
300	300	3.5	245	360	3.56
400	400	4.54	310	500	4.57

RESULTS

A_sB_s	A_xB_x	$\frac{A_sB_s}{A_xB_x}$	$\overline{\frac{A_sB_s}{A_xB_x}}$	R_x	R_s	x
30400	48400	.629	.791	8.02	10.12	-40.98'
14300	22100	.647	.801	8.14	10.14	-26.00'
3540	6110	.579	.759	7.15	10.22	- 9.6 '
2100	3400	.617	.784	8.04	10.26	+19.8 '
			Core Magnetized			
1500	2250	.666	.815	8.36	10.29	-13.4'
8000	14700	.544	.736	7.55	10.23	-106.4'
18000	28500	.631	.795	8.10	10.17	-16.6'
23800	35000	.680	.824	8.39	10.16	- 0.2'
33000	51200	.644	.801	8.15	10.14	-298.8'
39600	64400	.615	.784	7.96	10.12	-344.8'
50000	76500	.654	.809	8.20	10.12	- 27.7'
54600	86400	.631	.794	8.06	10.11	- 30.5'
65600	105000	.624	.790	8.01	10.10	- 27.16'
75000	120000	.625	.790	8.00	10.09	- 36.4 '
			Core magnetized $\theta = 60^\circ$			
6000	8500	.706	.840	8.65	10.23	- 40.8'
18750	32000	.584	.764	7.75	10.15	- 19.1 '
37400	68000	.550	.740	7.50	10.13	- 19.7 '
73500	108000	. 68	.824	8.35	10.12	- 28.2 '
121000	200000	.604	.776	7.85	10.10	- 9. '

RESULTS

$A_s - A_x$	$B_s - B_x$	$R_x - R_s$	$2A_x$	$2B_x$	R_s
- 92	- 2	- 2.10	504	384	10.12
- 60	0	- 2.00	340	260	10.14
- 48	0	- 2.47	180	136	10.22
- 26	0	- 2.22	136	100	10.20
5	-20	- 1.93	90	100	10.29
5	-60	- 2.68	210	280	10.23
0	-70	- 2.07	300	380	10.17
- 5	-60	- 1.77	350	400	10.16
- 6	-15	- 1.99	410	500	10.14
-10	-100	- 2.16	460	560	10.12
- 5	-100	- 1.92	510	600	10.12
-10	-110	- 2.05	540	640	10.11
-20	-115	- 2.09	600	700	10.10
-15	-130	- 2.09	630	760	10.09
15	- 40	- 1.58	170	200	10.23
- 10	- 75	- 2.40	320	400	10.15
- 15	-120	- 2.63	470	580	10.13
0	-115	- 1.77	600	720	10.12
- 10	-190	- 2.25	800	1000	10.10

Experiment No. 7

Wave Form and Iron Losses in Transformers

Introduction

The rapid development of power transmission by means of a step up transformer at the generator end and a step down transformer at the receiver end has brought the alternating current transformer into prominence in the electrical world. It is undoubtedly the most common and most used piece of alternating current apparatus on the market; on account of its unlimited use the losses must be kept at a minimum in order to keep the efficiency at a maximum and thus, this experiment has been performed and written up to show what effect the wave form has on the iron losses in a transformer.

Experiment No. 7

Wave Form and Iron Losses in Transformers.

The transformer used in this experiment was a 5.K.V.A. Westinghouse core type transformer with a 10:1 ratio. The generator used was a 30 K.W. alternating current generator which gave approximately a sine wave of voltage. The scheme of connections for this experiment is shown in fig. 7-A. The voltage taken from the generator is at a pressure of 110 volts and by means of an auto-transformer the voltage is varied from 110 to 440 volts at the generator end. The voltage over the primary of the transformer is maintained constant at 110 volts by means of a series resistance in the circuit. An ammeter measures the magnetizing current and a wattmeter measures the iron loss at the various generator voltages. By varying the voltage from 110 to 440 a wave shape varying from an approximate sine wave to a decided peak wave may be obtained. The following data was obtained.

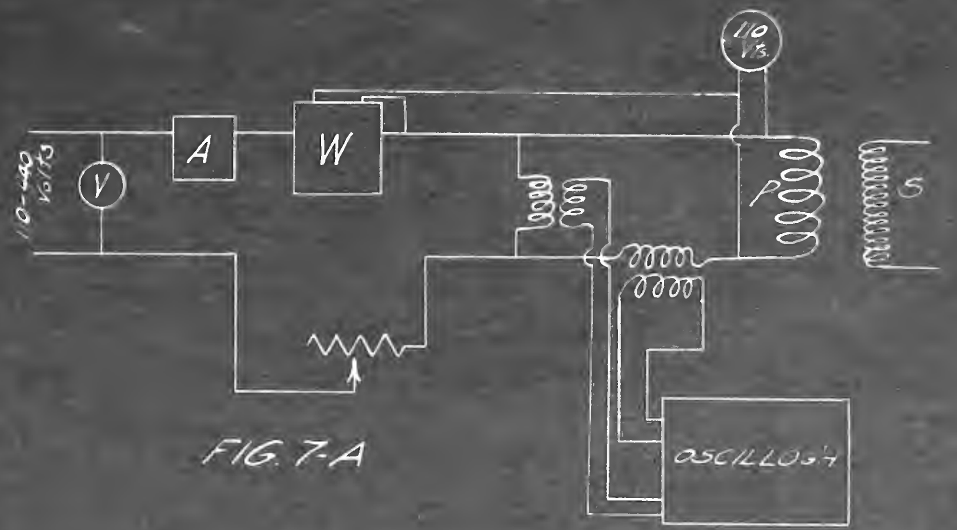
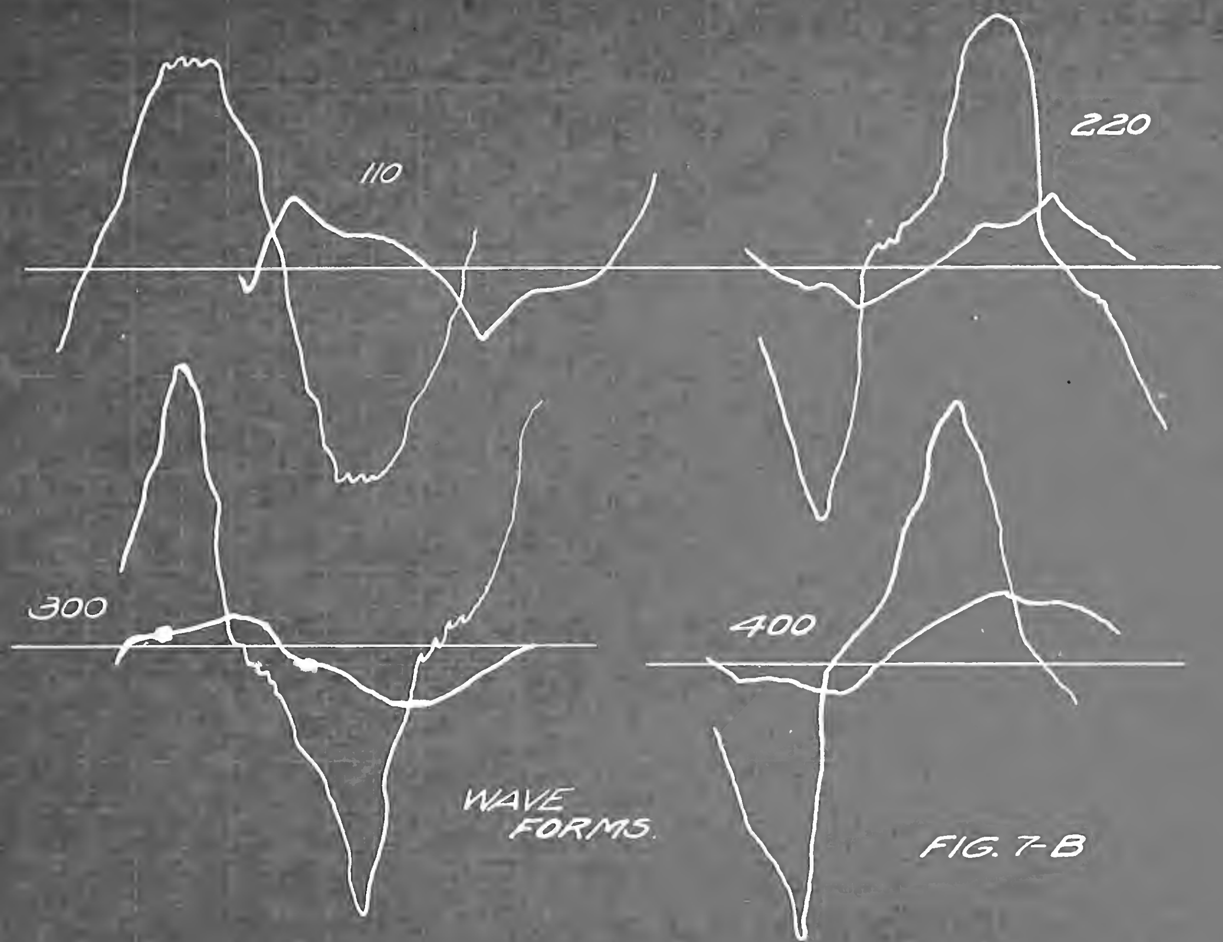


FIG. 7-A

Watts	Primary Current	Generated Voltage	Primary Voltage
125	2.30	110	110
120	1.58	170	110
115	1.66	220	110
113	1.60	270	110
109	1.56	336	110
106	1.52	440	110

The wave shapes obtained on the oscillograph for the current and voltages in the transformer at generated voltages of 110 volts, 220 volts, 300 volts and 400 volts are shown in fig. 7-B. At 110 volts the voltage is approximately a sine wave while at 400 volts it has a very decided peak to it. The current curve on the other hand is peaked at 110 volts and approaches a sine wave at 400 volts.



WAVE FORMS.

FIG. 7-B

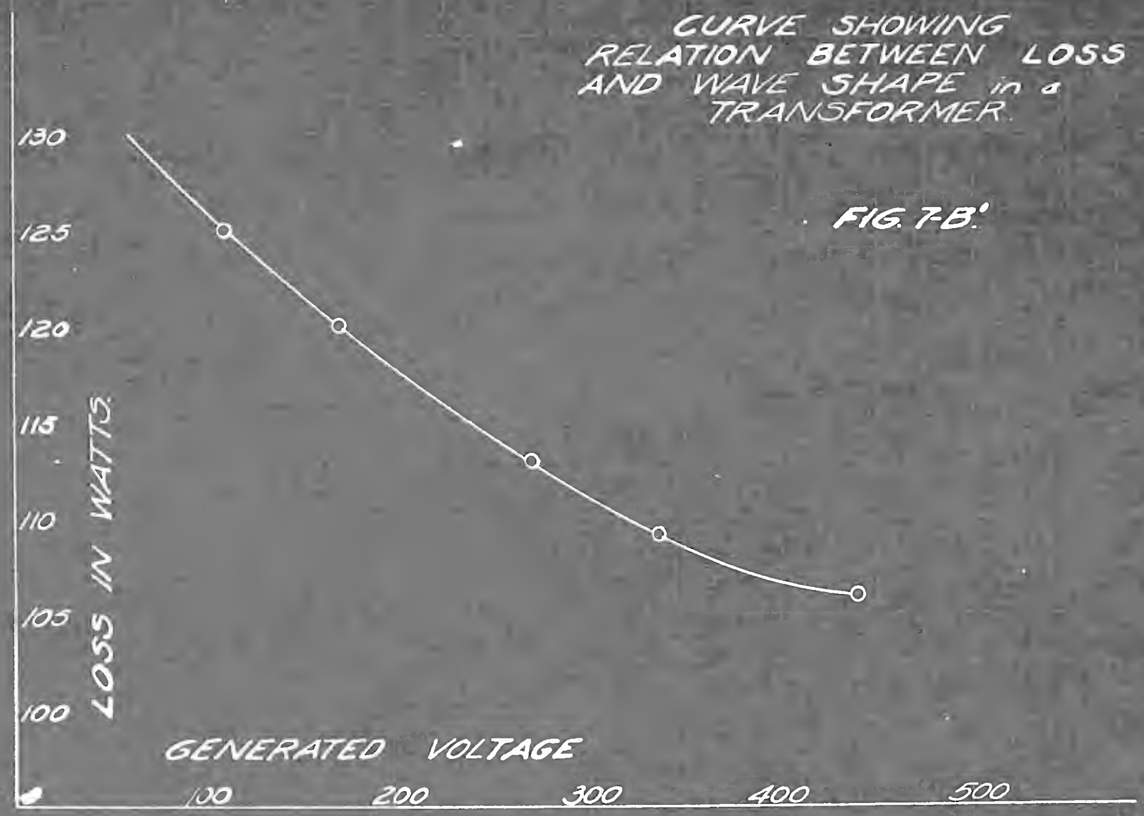

CURVE SHOWING
RELATION BETWEEN LOSS
AND WAVE SHAPE in a
TRANSFORMER
LOSS IN WATTS
130
125
120
115
110
105
100
GENERATED VOLTAGE
100
200
300
400
500

FIG. 7-B'

Discussion.

From the data it will be noted that the higher the peak of the voltage wave the less the loss in the transformer at the same effective voltage or in other words the greater the form factor the less the iron loss. The reason for this is as follows: The effective value of the induced e.m.f. is given by the equation $E = 4fBSFN$ where E is equal to the effective value of the e.m.f., f the frequency, B the maximum flux density, S the cross sectional area, F the form factor and N the number of turns. From the above equation it may readily be seen that $B = \frac{E}{4fSFN}$ which shows that the maximum flux density for a given electromotive force is inversely proportional to the form factor. Therefore for flat curves with a small form factor the iron losses are larger than for pointed waves which have a large form factor.

The mean square of any wave is the sum of the mean squares of all its harmonic components and consequently its effective value is independent of the phase relations of these harmonics. An e.m.f.

wave being a symmetrical one contains only odd components. If one of these has the same sign as the fundamental near the center of the latter, it will add to the peakness of the wave, but at the same time it will reduce the mean value since it will have one more negative than positive half waves included in the positive half of the given wave. On the other hand if the sign of this harmonic is reversed it will flatten the wave and increase its mean value. Consequently since the form factor is the effective value divided by the average value and since the effective value is the same in both cases, the peaked wave will have a greater form factor. For equal primary potential the primary current and power decrease with increase of form factor and the efficiency is thereby greater with a pointed potential wave. The regulation of the secondary potential, however, is better with a sine potential curve.

The advantages of a high form factor are counterbalanced by disadvantages. A peaked wave

takes a larger charging current on long transmission lines than one of sine curve form. Also the insulation must be made to withstand greater voltages on a peaked potential wave circuit than a sine potential wave, both having the same effective value. Take for instance two waves with effective values of 5000 volts, one of rectangular and the other of triangular form. Their form factors are 1 and 1.16 respectively and their amplitudes 5000 and 86660 volts respectively. This while the insulation of the first transformer has only to withstand 5000 volts, that of the other must be made for 8660 volts. The waves considered have extreme values and the ratio between the maximum and effective values generally lies within these limits. When measuring the peak voltage of a transformer it is absolutely necessary to measure the exact peak value and not the effective. This point may be noted from the former curves. At 110 volts and 400 volts, the effective values are the same

but the peak values are different so that the effective value multiplied by the square root of 2 is not a correct method to obtain the maximum voltage. There are three ways that may be used to measure the true peak voltage: the spark gap, the oscillograph and the crest voltmeter. The spherical spark gap voltmeters are constructed of two non arcing metal spheres mounted vertically in suitable wooden frame work. One sphere is sometimes used at ground potential or both may be insulated as the case may be. A resistor is placed in series with one of the spheres to protect the transformer when the spark jumps over. Suitable micrometer adjustment is provided so that the separation of the spheres may be accurately determined. The oscillograph shows a band of light on a screen and the length of this band is proportional to the peak voltage.

The crest voltmeter operates on the principle that the average value of the half wave of the charging current in a condenser bushing which

charging current flows into and out of the bushing when the latter is subjected to a voltage strain, is proportional to the crest of the voltage wave. The charging current is rectified by means of small mercury bulbs and measured by a sensitive direct current milli-ammeter that is calibrated in terms of the high alternating voltage. The deflection of the instrument is proportional to the maximum value of the voltage wave; and since the instrument is a direct current instrument, all the scale divisions are approximately equal. This makes it possible to read low voltages on the scale with the same accuracy as the higher voltages, a very important and desirable feature in insulation testing. The readings of the crest voltmeter are affected by the frequency of the supply circuit and for this reason a frequency meter is supplied. The readings of the crest voltmeter vary directly with the frequency so that a correction can basily be made for any variation in frequency from that for which it is calibrated. The complete equipment

of the crest voltmeter output consists of a small slate panel mounted on pipe framework and upon the panel is mounted a frequency meter, a direct reading highly sensitive milli-ammeter specially calibrated to read crest voltages, two small rectifier bulbs and a change over switch.

When testing a transformer on non sine waves, reliable results may be obtained by the following scheme (fig. 7-C). If the alternator has a frequency of 60 cycles a circuit containing resistance, inductance and capacity is so adjusted that it will be in resonance at 60 cycles and then by taking leads off at each end of the resistance and using low values of voltage and current as compared to those of the main circuit, a sine wave circuit may be thus obtained. Another method for measuring losses with non-sine wave is to use an iron loss voltmeter which is described as follows: The iron loss voltmeter is for use with a wattmeter for determiningthe iron loss in transformers on the basis of sine

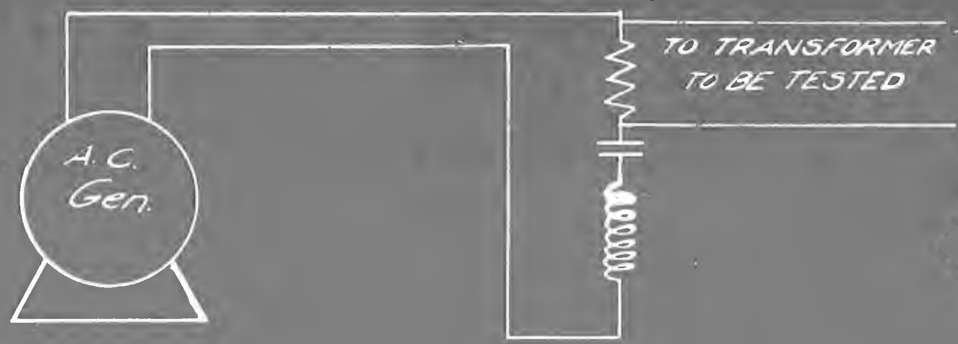


FIG. 7-C.

wave voltage and normal frequency, when the testing is done on a circuit of any wave shape and approximately normal frequency. The iron loss voltmeter consists of a wattmeter movement connected in series with the winding on an iron core, and so compensated that it measures the iron loss in the core. It is calculated in volts on a circuit of pure sine wave voltage. Any circuit that makes the instrument read a certain voltage therefore produces the same iron loss as would a pure sine wave of that voltage. The instrument does not indicate the voltage of the circuit, but the voltage of a pure sine wave of normal frequency that would cause the same iron loss in the transformer as the wave of voltage of the testing circuit. In application the iron loss voltmeter is connected across the terminals of the transformer under test in the same manner as an ordinary voltmeter. A wattmeter is also connected in circuit in such a way as to measure the total input of both transformer and iron loss

voltmeter. The voltage of the circuit is then adjusted by any convenient means until the iron loss voltmeter reads the normal voltage of the transformer. The total power input is read on the wattmeter and the watt input of iron loss voltmeter is read on its watt scale, the difference being the normal iron loss of the transformer. This concludes the work of experiment number 7 on "Wave Form and Iron Losses in Transformers".

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The Efficiency of Three Phase Transformers by the Opposition Method

INTRODUCTION

In the opposition method the only energy supplied to the transformer under test is that necessary to overcome the losses. This method is therefore a very convenient and economical way to make heat runs on large transformers for were the full load to be carried during the whole test the performance would be an expensive one.

DETAILS OF THE METHOD

In testing one three-phase transformer a single phase current of normal rated value, 4.54 in our test, is sent through the high tension windings connected in series as shown in Fig. 1. The low tension side is connected delta to a three-phase supply. For regulating the supply current in the high tension winding a common lamp rack may be used. Two wattmeters, or one wattmeter and a selective switch are needed on the three-phase side and one wattmeter on the single phase side to measure the iron and copper loss respectively.

When two transformers are to be tested by pairing the high tension windings are star connected and normal current sent through each winding. (Since the separate coils are all in parallel three times the normal rated current must be fed to the neutrals).

A three-phase e.m.f. of normal value is impressed on the low tension side delta connected and paralleled. The measuring instruments are arranged as in the testing of the one transformer. See Fig. 2.

THEORY OF THE METHOD

Referring to Fig. 1, were the leads of the high tension side to be disconnected,the difference of potential between a and b would be 0. Hence the wattmeter in the low tension side would indicate the iron losses.

If a single phase current equal to the rated load is sent through the high tension side a current is caused to flow in the closed delta of the low tension side. The e.m.f. induced is not great

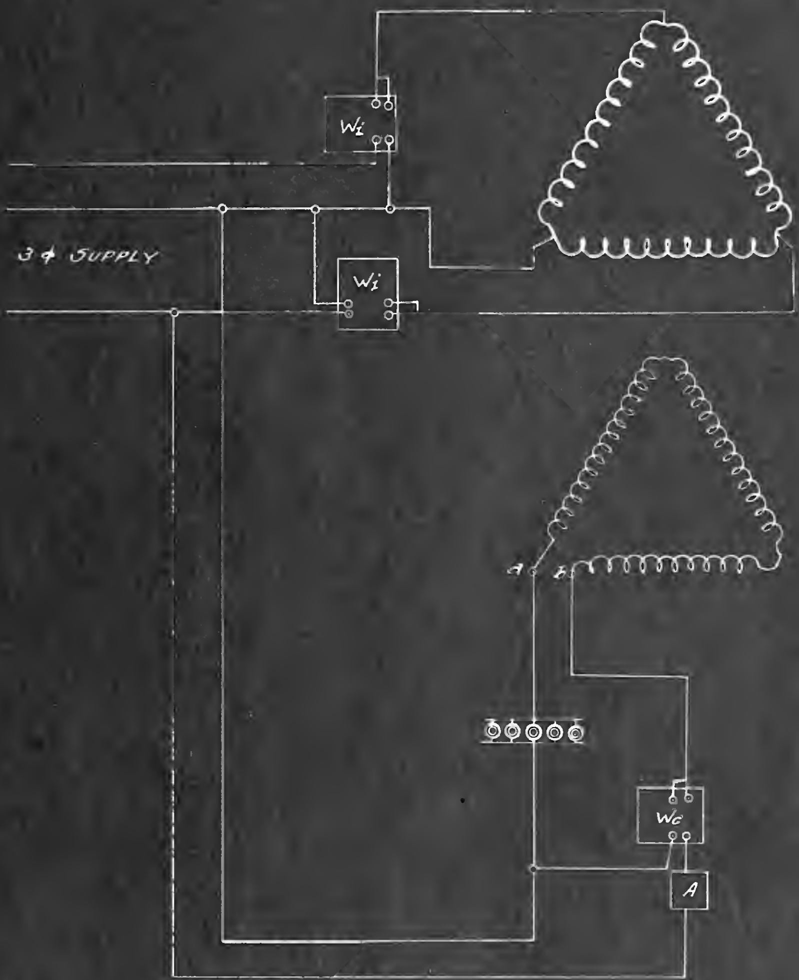


FIG. 1

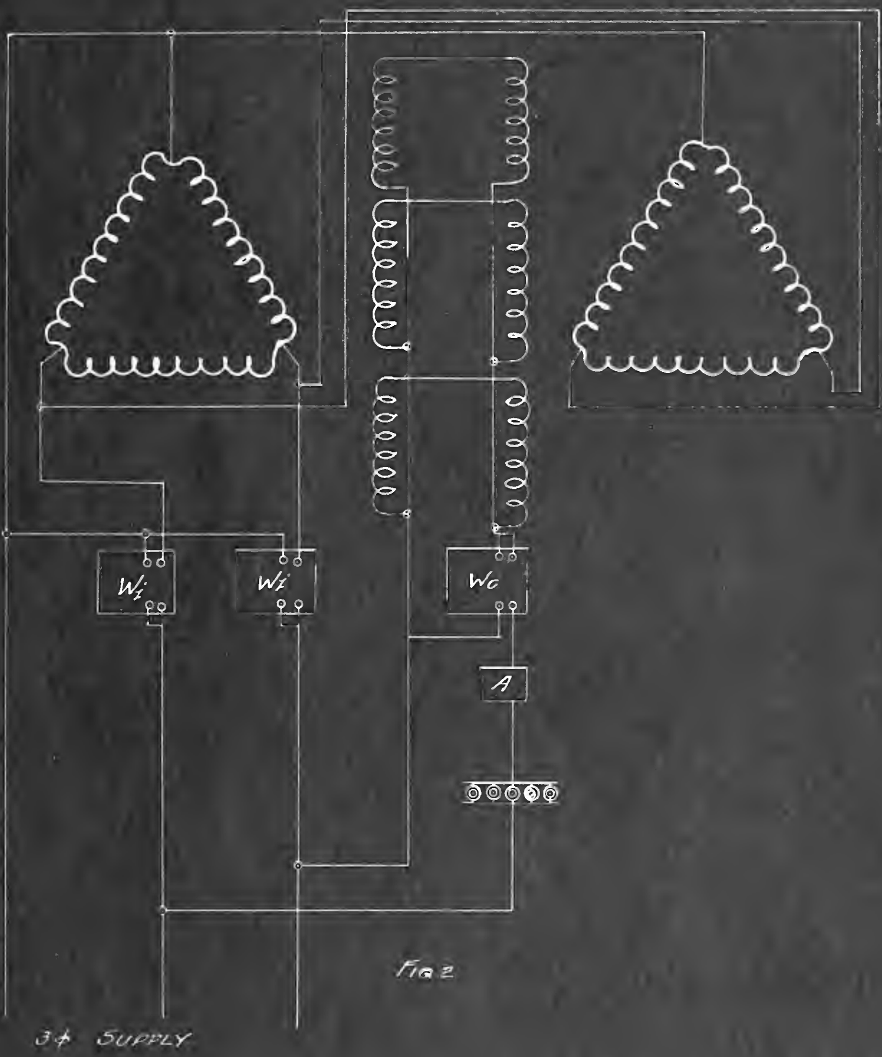


Fig 2

enough to send the current out of the delta. Hence all the energy supplied to the high tension side is the total copper loss.

Likewise, if the transformers are paired as shown in Fig. 2 and three times the rated current be sent through the high tension sides a similar action takes place. The energy supplied to the high tension sides is then a measure of the copper loss while that supplied to the low tension sides is a measure of the iron loss.

OBSERVATIONS

One transformer

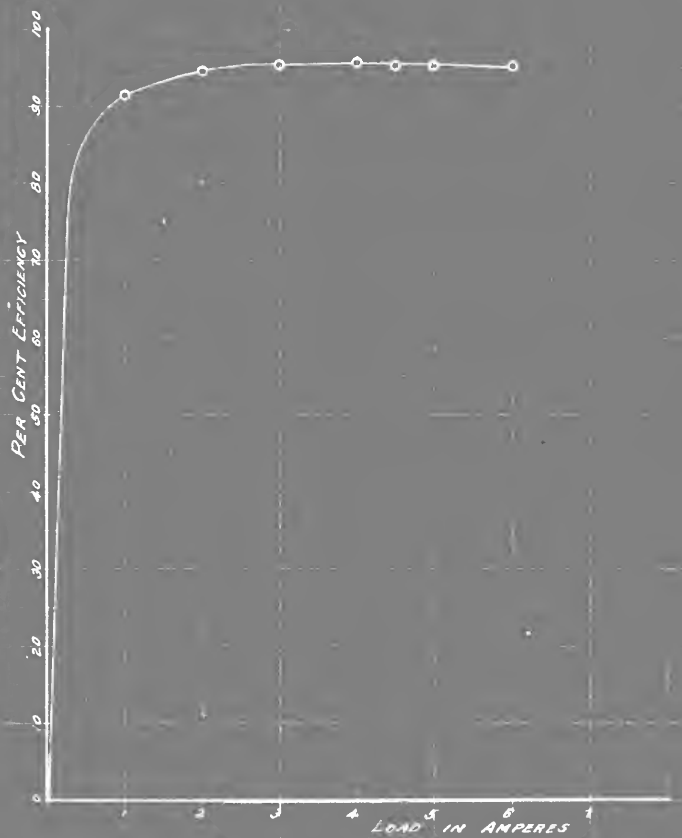
$I = 4.54$ $V = 110$ 25 cycles

Copper loss = 405 watts Iron loss = 300 watts.

Two transformers

$I = 13.62$ $V = 110$ 25 cycles

Copper loss = 810 watts Iron loss = 600 watts.



PER CENT EFFICIENCY
100
90
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
LOAD IN AMPERES

The Regulation of Alternators

INTRODUCTION

Thus far no method has been found whereby the regulation of an alternator can be determined with any degree of accuracy without loading the machine to its rated load. For obvious reasons this cannot always be done, therefore many methods of approximation have come into practice.

Out of the various methods of determining the regulation the American Institute of Electrical Engineers adopted the following three with preference in the order they are given:

Method A:- The regulation can be measured directly, by loading the generator at the specified load and power-factor, then reducing the load to zero, and measuring the terminal voltage, with speed and excitation adjusted to the same values as before the change. This method is not generally applicable for shop tests, particularly on large generators, and it becomes necessary to determine the regulation from such other methods of testing as can be readily made.

FIG 1.

FIG. 2

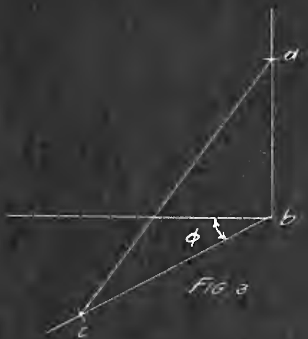


FIG 3

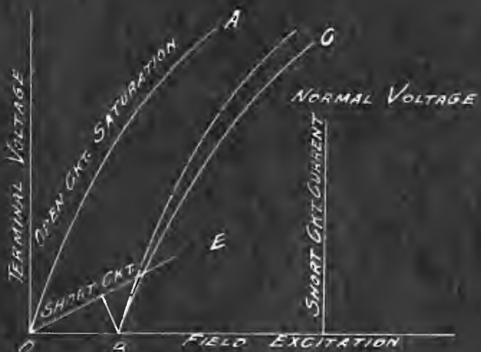


FIG. A

Method B:- This consists in computing the regulation from experimental data of the open-circuit saturation curve and the zero power factor saturation curve. The latter curve, or one approximating very closely to it, can be obtained by running the generator with over-excited field on a load of idle-running under-excited synchronous motors. The power-factor under these conditions is very low and the load saturation curve approximates very closely the zero-power factor saturation curve. From this curve and the open circuit curve points for the load saturation curve, for any power-factor, can be obtained by means of vector diagrams.

To apply this method, it is necessary to obtain from test the open circuit saturation curve, OA and the saturation curve OB at zero-power factor and rated load current. At any given excitation Oc, the voltage that would be induced on the open circuit is ac, the terminal voltage at zero-power factor is bc, and the

apparent internal drop is ab. The terminal voltage dc, at any other power-factor can be found by drawing the e.m.f. diagram as in Fig. 2, where ϕ is an angle such that cos ϕ is the power factor of the load, be the resistance drop (IR) in the stator winding, ba the total internal drop, and ac the total induced voltage; ba and ac being laid off to correspond with the values obtained from Fig. 1. The terminal voltage at power-factor cos ϕ, is then cb of Fig. 2, which, laid off in Fig. 1 gives point d. By finding a number of such points the curve Bdd' for power-factor for cos ϕ is obtained and the regulation at this power-factor (expressed in per cent.) is $\frac{100 \times a'd'}{d'c'}$, since a'd' is the rise in voltage when the load at power-factor cos ϕ is thrown off at normal voltage c'd'.

Generally the ohmic drop can be neglected, as it has very little influence on the regulation, except in the very low speed machines

where the armature resistance is relatively high,or in some cases where regulation at unity power-factor is being estimated. For low power-factors, its effect is negligible in practically all cases. If resistance is neglected, the simpler e.m.f. diagram, Fig. 3 may be used to obtain points on the load saturation curve for the power-factor under consideration.

Method C:- Where it is not possible to obtain by test a zero power-factor saturation curve as in the foregoing method, this curve can be estimated closely from open-circuited and short-circuited curves, by reference to tests at zero power-factor on other machines of similar magnetic circuit. Having obtained the estimated zero power-factor curve, the load saturation for any other power-factor is obtained as in the foregoing method.

Thus this method is the same as the method just described, except that the zero power-factor curve must be estimated. This may be done

as follows: In Fig. 4, OA is the open-circuit saturation curve and OE the short-circuit line as shown by test. The zero-power-factor curve corresponding to any given current BF will start from point B, and for machines designed with low saturation and low reactance, will follow parallel to OA as shown by the dotted curve BD, which is OA shifted horizontally parallel to itself by the distance OB. In high speed machines, or in others having low reactance and a low degree of saturation in the magnetic circuit the zero power-factor curve will lie quite close to BD, particularly in those parts that are used for determining the regulation. This is the case with many turbo-generators and high-speed water-wheel generators. In many cases, however, the zero-power-factor curve will deviate from BD, as shown by BC, and the deviation will be most pronounced in machines of high reactance, high saturation, and large magnetic leakage.

The position of the actual curve BC with

relation to BD can be approximated with sufficient exactness by investigating the corresponding relation as obtained by test at zero power-factor on machines of similar characteristics and magnetic circuit. Or curve BC can be calculated by methods based on the results of tests at zero power-factor. After BC has been obtained the saturation curve and regulation for any other power-factor can be derived as described in the second method.

CONDITIONS FOR TESTS OF REGULATION

Speed and Frequency:- The regulation of generators is to be determined at constant speed, and of alternating-current apparatus at constant frequency.

Power Factor:- In apparatus generating, transforming or transmitting alternating-currents, the power-factor of the load to which the regulation refers should be specified. Unless otherwise specified, it shall be understood as referring to non-inductive load, that is to a load

in which the current is in phase with the e.m.f. at the output side of the apparatus.

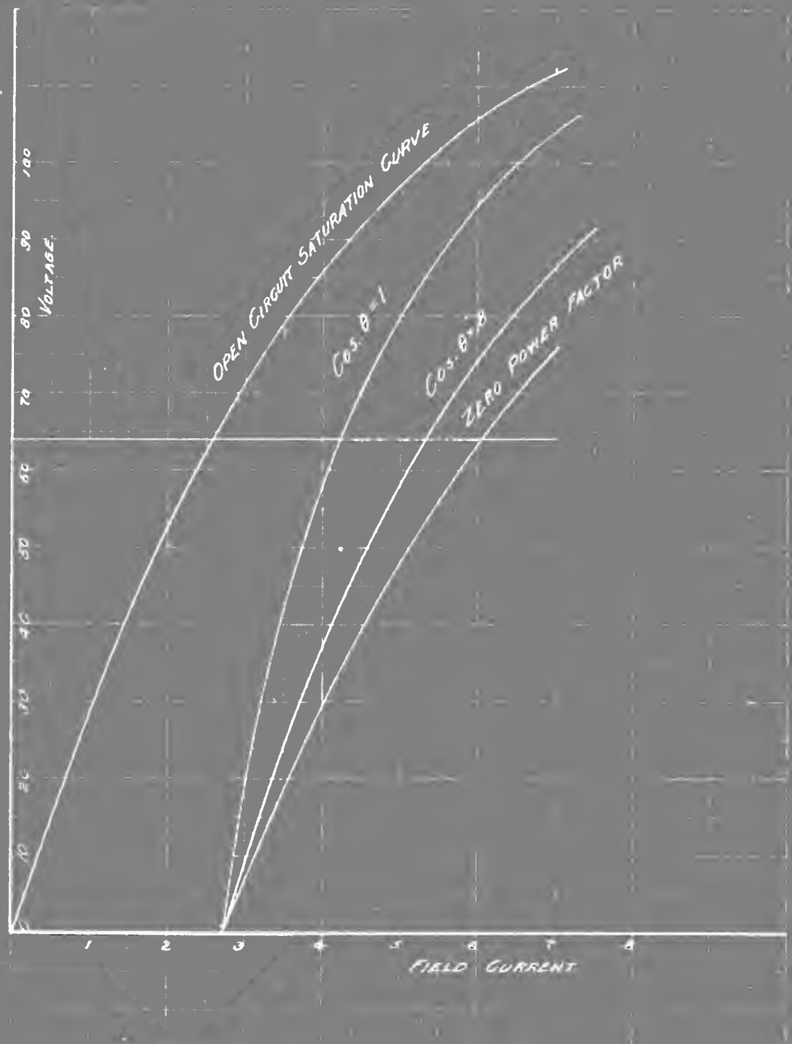
Wave Form:- In the regulation of alternating-current machinery receiving electric power, a sine-wave of voltage is assumed, except where expressly specified otherwise.

Excitation:- In commutating machines, rectifying machines, and synchronous machines, such as direct-current generators and motors, as well as in alternating-current generators, the regulation is to be determined under such conditions as to maintain the field adjustment constant at that which gives rated-load voltage at rated-load current, as follows:

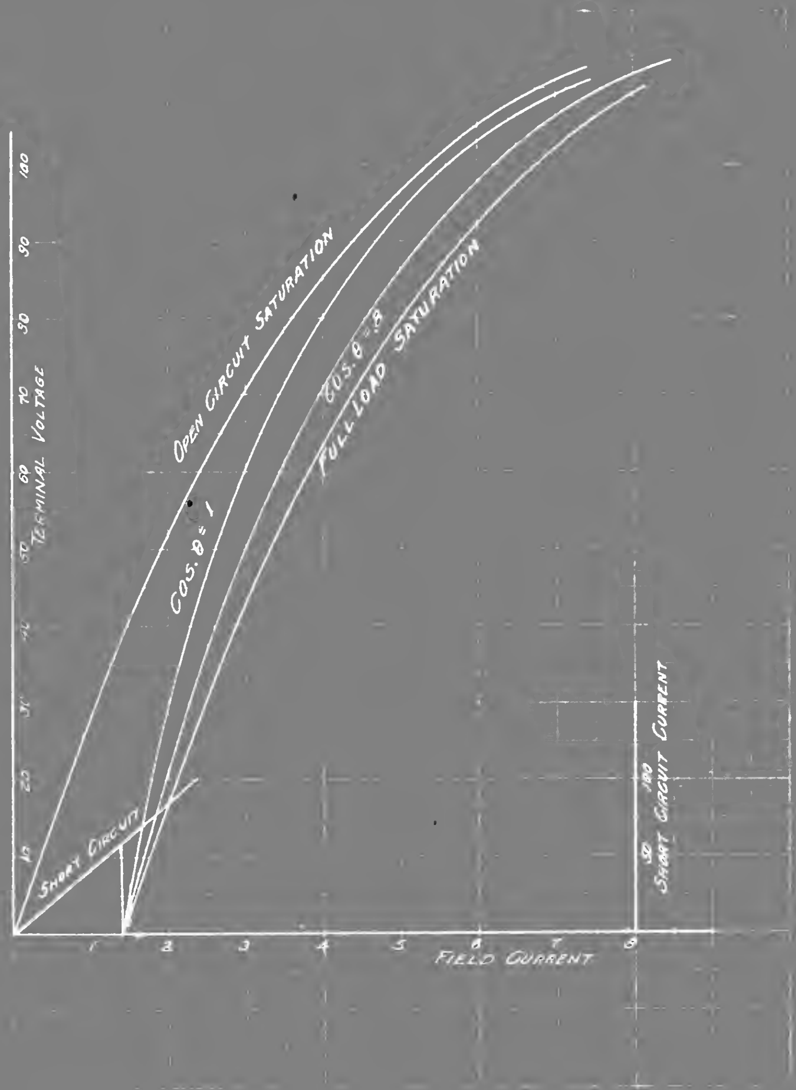
(1) In the case of separately excited field magnets - constant excitation.

(2) In the case of shunt machines, constant resistance in the shunt field circuit.

(3) In case of series or compound machines, constant resistance shunting the series-field windings.



Voltage
100
90
80
70
60
50
40
30
20
10
0
Open Circuit Saturation Curve
Cos. θ=1
Cos. θ=.8
Zero Power Factor
1
2
3
4
5
6
7
8
Field Current



Open Circuit Saturation
Cos. θ = 1
Cos. θ = .8
Full Load Saturation
Short Circuit
Terminal Voltage
Field Current
Short Circuit Current

OBSERVATIONS

Method A.

I_f	P.F.	Terminal Voltage	Frequency in cycles
6.89	.85	96	50
6.89		111	
3.02	.85	57	50
3.02		725	
3.1	1.00	59	50
3.1		72	
6.85	1.00	105	50
6.85		111	

Method B.

Open Circuit Saturation		Full Load Saturation	
I_f	E_t	I_f	E_t
5.1	96.3	5.1	98
4.9	94.2	4.5	93.2
4.5	91.	4.0	88.0
4.0	84.	3.5	82.0
3.5	76.5	3.0	75.0

Method B (continued)

Open Circuit Saturation		Full load Saturation	
I_f	E_t	I_f	E_t
3.0	69.0	2.5	68.0
2.0	46.5	2.0	59.
1.5	36.0	1.0	40.
1.0	23.0	.5	32.

Frequency 50 cycles.

RESULTS

I_f	P.F.	Per cent Regulation		
		Method A.	Method B.	Method C.
6.89	.85	15.7	29.	4.75
6.89	1.0	6.6	7.7	1.85
3.1	.85	26.2	600.	60.
3.1	1.0	22.0	204.	60.

EXPERIMENT 9.

STUDY OF IRON AND COPPER LOSSES IN TWO TRANSFORMERS

The usual methods for measuring losses in a transformer are: the open circuit method for iron losses; and the short circuit method for copper losses. The short circuit method for copper loss is not always convenient especially in testing large transformers where excessive current is required to fully load the transformer.

To overcome this difficulty the opposition method has been devised.

The object in performing this experiment was to determine how the losses when taken by the opposition method compare with the losses when taken by the short and open circuit methods; and also to note the effect on the results of changing the phase angle and frequency of the secondary current.

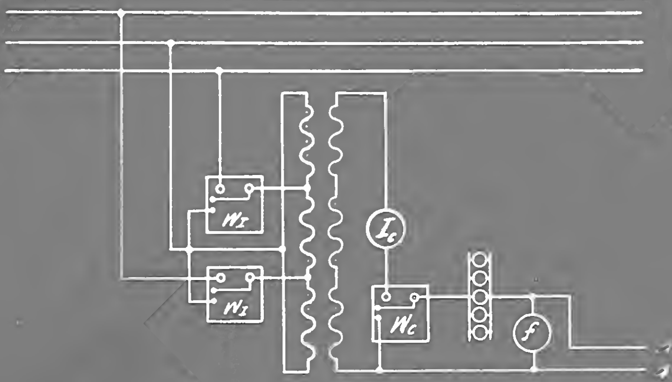


FIG. A

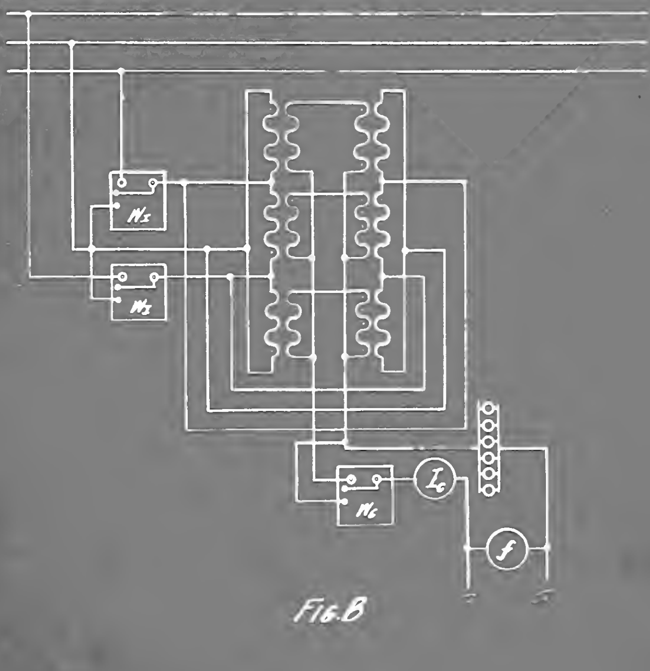


FIG. B

Transformers:

Two 3 phase, 25 cycle, 110 to 1100 volt G.E.

The first test was the opposition method applied to one 3 ∅, 5 K.V.A., 25 cycle transformer. The apparatus was first connected as shown in Fig. A, and the iron and copper losses were read on meters Wi and Wc respectively;the load current being adjusted for full value,viz., 4.54 amperes on the 1100 volt (high tension) side. Leads aa were first connected across one phase of the primary, and then successively to each of the other two phases, and to the first phase reversed. Next aa were connected to a separate source and the frequency varied in steps of 10 cycles from 60 cycles down to 20 cycles. For each arrangement the iron loss and copper loss was noted. The load current was kept as near 4.54 amperes as possible.

The second test was the opposition method applied to two 3 ∅, 5 K.V.A., 25 cycle transformers paired. The apparatus was connected as

in Fig. B ; and the iron and copper losses were noted.

The third test was the measurement of the copper loss by the short circuit method, and the iron loss by the open circuit method. These determinations were made for each of the transformers.

Opposition Method on
1 Transformer -
Changing source of I_c

	I_c	W_i	W_c	W_c correct
Load Current				
1st phase	4.32	315.	393	431
2nd phase	4.31	317.5	393	435
3rd phase	4.3	307.5	387	430
1st	4.32	300.	398	438

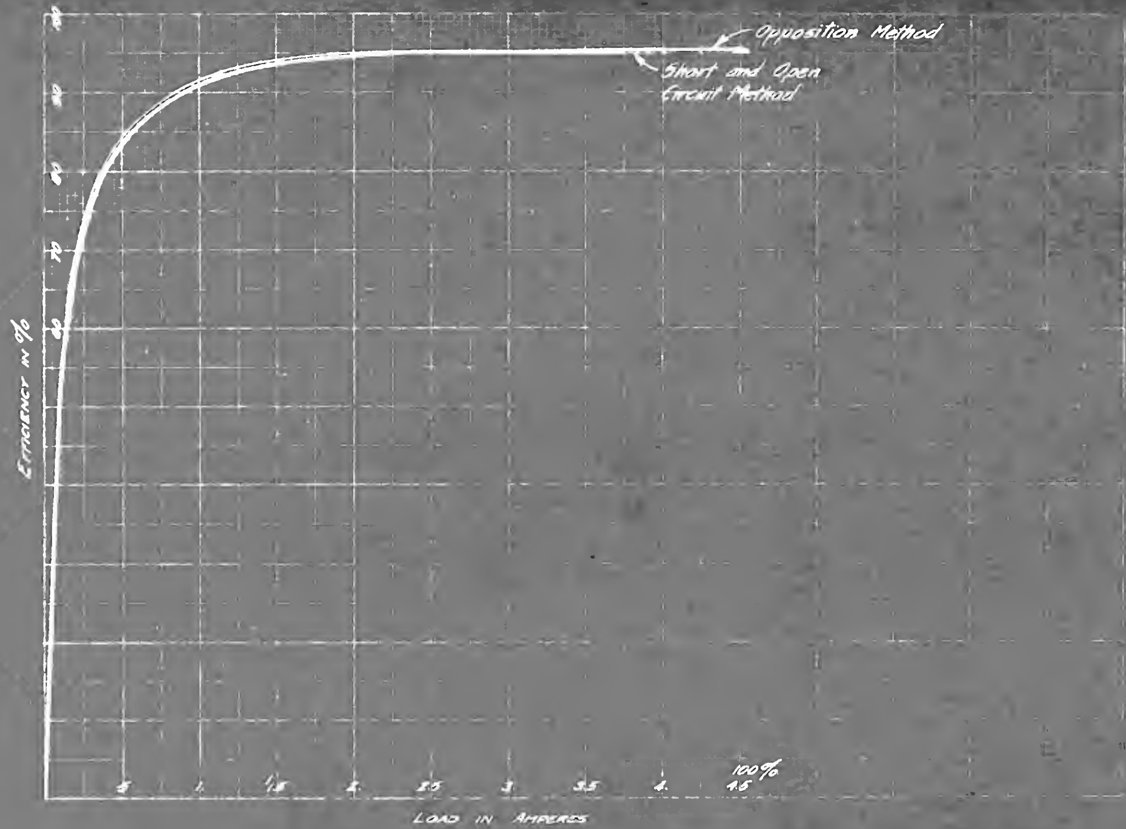
Opposition Method on
1 Transformer - Varying
Frequency I_c

	I_c	W_i	W_c	W_c correct	f
Load Current					
1st phase	4.58	312.5	490	480	60
2nd phase	4.54	325.	478	478	50
3rd phase	4.	325.	347	446	40
1st	3.55	320.	268	437.5	30

	Short and open circuit Method			Paired Opposition Method				
	I_c	W_i	W_c	I_c	W_c	W_i	W_i/T	W_c/T
East	13.62	303	474	13.62	654	860	324	430
West	13.62	287	470					

Efficiency

	Opp.	Short & Open
Load	% Eff.	% eff.
4.54	95.2	95.1
4.	95.3	95.1
3.5	95.1	95.1
3.	95.1	95.
2.5	94.9	94.9
2.	94.4	94.45
1.5	93.3	93.4
1.	90.8	90.9
.5	83.7	84.4
.25	68.05	68.95
.1	51.15	52.1



Opposition Method
Short and Open Circuit Method
Efficiency in %
Load in Amperes
100%
.5
1.
1.5
2.
2.5
3.
3.5
4.
4.5

RESULTS

The data obtained shows that varying the phase angle of the load current effects the copper loss by about 2% and the efficiency by about .1%.

Changing the frequency of the load current, however, effects results to a certain degree; the copper loss decreases with a decrease in frequency.

In the opposition method the copper losses are slightly smaller and the iron losses slightly larger than in the short circuit and open circuit method. The two discrepancies seem to balance each other as the efficiency curves by the two methods are almost identical.

The efficiency was calculated from the equation:

$$\text{Eff.} = \frac{\text{output}}{\text{output} + \text{copper loss} + \text{iron loss}}$$

I_c assumed to be 4.54 amperes per phase.

The opposition method has two great advantages:

1. Iron loss and copper loss are measured simultaneously, and without changing connections.

2. The only energy consumed is that which is necessary to supply the losses. This method is, therefore especially adapted to long heat tests, where the energy consumed is a large item.

EXPERIMENT NO. 10

- A Study of a Certain Condenser. -

Experiment No. 10

A Study of a Certain Condenser.

Object:

It is a well known fact that the impedance of a condenser is inversely proportional to the frequency. Also, by Ohm's law, the current is inversely proportional to the impedance.

Combining the above two statements we get a final expression by which, the current is directly proportional to the frequency.

In an experiment on study of Inductance and capacities performed by the senior class of Armour Institute, it was found that a certain condenser showed increase of current with increase of frequency up to a certain point, and then a decrease of current with a still further increase of frequency.

It is the purpose of this experiment, therefore, to show why this apparently abnormal behavior takes place.

A Study of a Certain Condenser.

INTRODUCTION

Before we take up the discussion of characteristics of our condenser, perhaps it will be worth while to give in detail the definition of:

Resistance,
Inductance and
Capacity;

also the effects of placing either one in a circuit, and finally the general characteristics of each one separately.

RESISTANCE

Resistance is the opposition offered to the passage of current in a conductor.

It is the ratio of $\frac{\text{Volts}}{\text{Amperes}}$ for a Direct-Current circuit.

If placed in a circuit it does not produce any phase displacement of voltage and current, but simply reduces the value of current, the

e.m.f. remaining constant; in other words the resistance, or R I drop is always in phase with the current.

In an A-C circuit it is usually called effective resistance, and it is equal to the ratio of the total power as indicated by a watt-meter, to the Voltage of the circuit.

INDUCTANCE

The number of interlinkages of an electric circuit with the lines of magnetic force of the flux produced by unit current in the circuit is called inductance of the circuit. It is the ratio of the induced e.m.f. to the ratio of change of current.

$$e = - \frac{d\phi}{dt} \quad K$$

$$L = - \frac{e}{\frac{di}{dt}} = \frac{d\phi}{dt} \cdot \frac{dt}{di} = \frac{d\phi}{di}$$

Inductance can be either

(1) Self inductance = L as defined above.

(2) Mutual Inductance = M;

Mutual Inductance is the number of inter-linkages of an electric circuit with the lines of magnetic force of the flux produced by unit current in a second electric circuit.

$$M^2 = \text{ or } < L_1 L_2$$

It is equal if all of the flux is inter-linked with both circuits, and smaller if not. The practical unit of inductance is the henry (h) and the milhenry (m h).

In an A-C circuit the instantaneous values of the current and voltage can be expressed by equations (1) and (2) respectively.

$$i = I_{max} \sin 2 \Pi ft \text{------------------------} (1)$$

$$e = - L \frac{di}{dt} = - L\, I_{max}\, 2 \Pi f \cos 2 \Pi ft. \quad (2)$$

Equation (1) is a sine wave while equation (2) is a cosine (see figure 10-A).

It is apparent, therefore, that the current and voltage are 90 degrees out of phase, as a result of which the power per cycle is equal to zero. (See figure 10_B.)

$$P = E\, I \cos \theta = E\, I \cos 90^{\circ} = 0$$

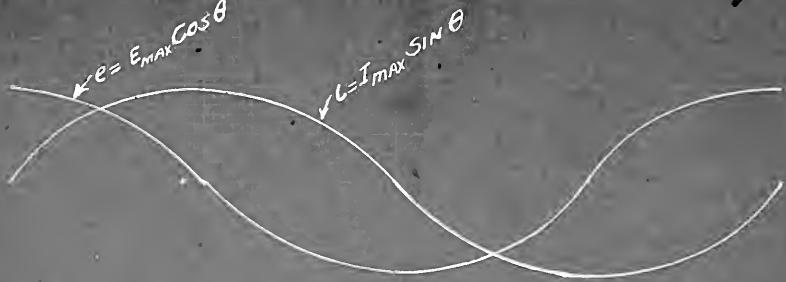


FIGURE 10A

FIGURE 10B

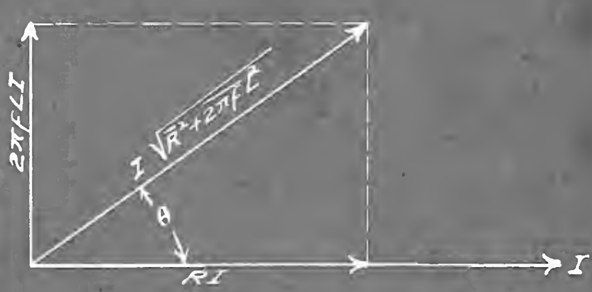


FIGURE 10C

The above relation is true provided the circuit we are dealing with is purely inductive which is practically impossible, because there is always some resistance in the circuit. Consequently figure 10-B must be slightly modified as shown by figure 10-C. Here the resultant voltage is equal to the vector sum of voltage consumed by the resistance = R I and the voltage consumed by the inductance = 2 π F L I.

$$\text{or} \quad E = I \sqrt{r^2 + \overline{2 \pi f L}^{\,2}}$$

$$= I \sqrt{r^2 \quad x\ 2}$$

$$= I \times Z$$

where Z is called the impedance of the cirucit.

The 2 π f L I component of the voltage is called the reactive, or wattless, while the R I, or resitance is called the power component. θ is the angle by which, (assuming counter-clock wise rotation) the current lags behind the e.m.f.

$$\theta = \tan^{-1} \frac{2 \pi f L}{R} = \tan^{-1} \frac{X}{R}$$

= phase angle of the circuit.

CONDENSERS AND CAPACITY

The definition of capacity in general is simply the ability of any piece of apparatus to store up energy in an electric field.

The charge of an electric condenser is proportional to the impressed voltage, that is potential difference at its terminals, and to its capacity.

A condenser is said to have a unit capacity if unit current existing for one second produces unit difference of potential at its terminals.

The practical unit of capacity is the farad - one farad is an extremely large unit, and, therefore, one millionth of one farad, called microfarad (m f) is commonly used.

If an alternating e.m.f. is impressed upon a condenser the charge of the condenser

varies proportionally to it, and thus there is current <u>to</u> the condenser during rising, and <u>from</u> the condenser during decreasing e.m.f.; that is the current taken by the condenser leads the impressed voltage by 90 degrees, or a quarter of a cycle,

Denoting

f = frequency

E = effective voltage impressed on condenser

C = Capacity in m f s.

The time of one complete charge or discharge is $\frac{1}{4f}$, since the condenser is charged and discharged twice each cycle.

Now since

$\sqrt{2}$ E is the maximum voltage impressed upon the condenser, and average of C E $\sqrt{2}$ 10^{-6} amperes would have to exist during one second to charge the condenser to this voltage, and to charge it in $\frac{1}{4f}$ seconds an average current of 4fC E $\sqrt{2}$ 10^{-6} amps is required.

Since $\frac{\text{EFFECTIVE I}}{\text{AVERAGE I}} = \frac{\pi}{2\sqrt{2}}$

for a sine wave, it follows that effective $I = 2 \pi f C E 10^{-6}$ amps - in other words at an e.m.f. E effective volts, and frequency f, a condenser of C m.f.s, capacity takes $I = 2 \pi f C E 10^{-6}$ amps.

The effective current leads the terminal voltage by 90° or by a quarter of a cycle.

Again the above 90 deg. assumption is not exactly true because due to the energy loss in the condenser by dielectric hysteresis and leakage, the current leads the e.m.f. by somewhat less than 90 deg., and therefore, it can be resolved into the wattless charging current and dielectric hysteresis plus leakage.

Finally then the current

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{10^6}{2\pi f C}\right)^2}} = \frac{E}{\sqrt{R^2 + X^2}} = \frac{E}{Z}$$

where

C = capacity in m f s

Z = impedance in ohms

$\frac{10^6}{2\pi f C} = X$ = condensive reactance.

Generally R is small compared with X and it can be neglected then:

$$I = \frac{2\pi f C E}{10^6} \qquad \text{--------} \ (A).$$

For an ideal condenser a curve plotted between (f) and (I) with constant (E) of sine wave is a straight line showing that I varies directly with the frequency.

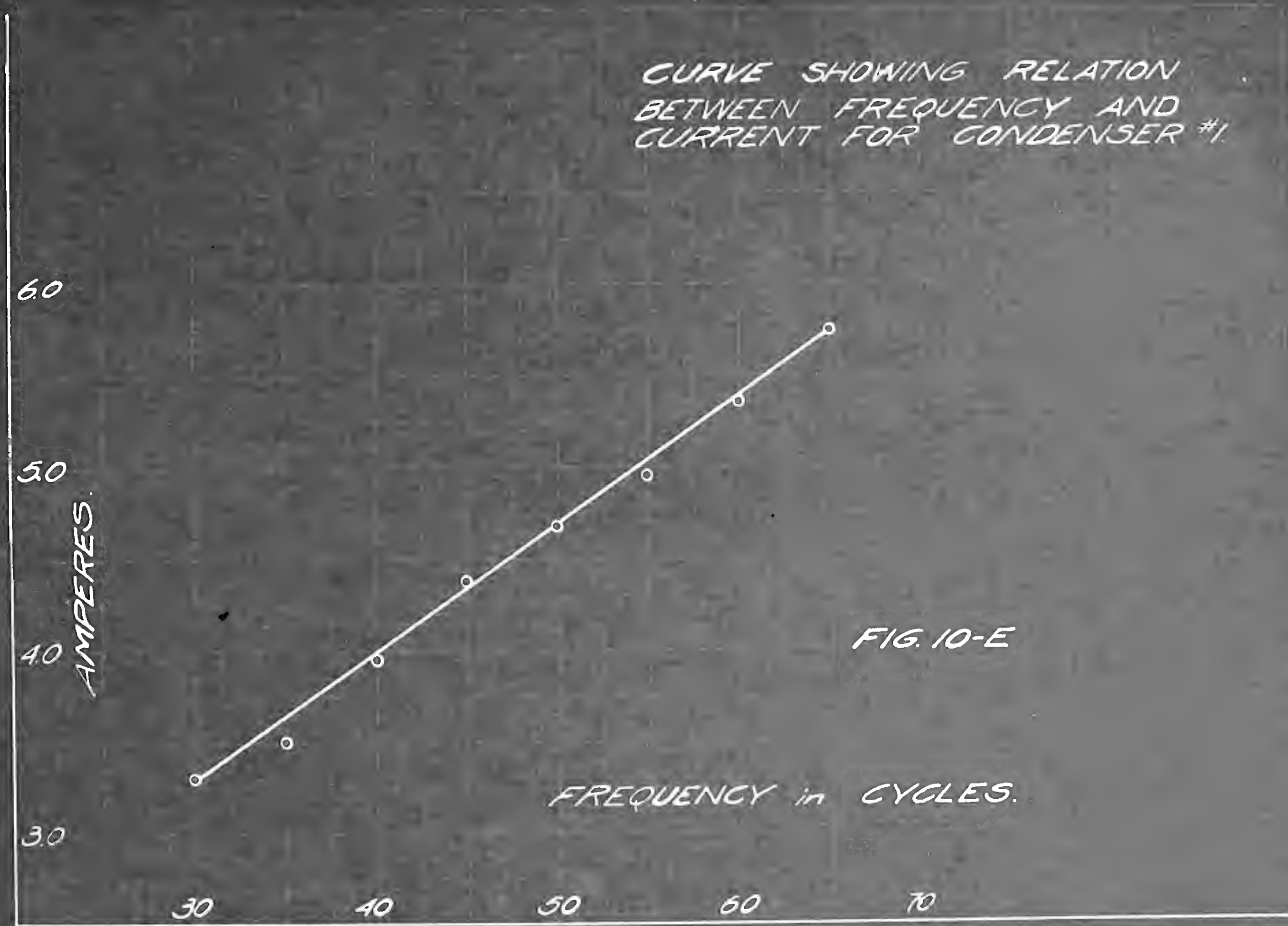
To prove the above theorectical assumption, 3 types of condensers were tested as follows:

The condensers used in this experiment were of two types; in one type the tin foil is soldered together at the ends and the two ends are used as the terminals, in other words, it consists of several small condensers in parallel; in the other type the tin foil is not joined at the ends and the condenser is of a flat spiral form. The Voltage across the condenser is maintained constant and the frequency varied from 30 to 65 cycles. In two of the condensers the current increased directly as the frequency, while in the third condenser the current curve had the shape shown in fig. 10-D. This is the current curve of the flat spiral formed condenser. The current first rises with an increase in

frequency, then decreases and with a still further increase in frequency the current rises again. This irregularity is evidently due to harmonics and to prove this, and inductance was placed in series with the condenser in order to damp out the higher harmonics and then the current curve varied directly with the frequency. The data taken on condenser No. 1 (oblong condenser with knife switches) are as follows:

Current in amps.	Frequency in cycles	Voltage
5.75	65	110
5.35	60	110
4.95	55	110
4.68	50	110
4.38	45	110
3.95	40	110
3.50	35	110
3.31	30	110

A curve showing the relation between the current and the frequency is shown in fig. 10-E. The data for condenser No. 2, known as the Marshall



CURVE SHOWING RELATION BETWEEN FREQUENCY AND CURRENT FOR CONDENSER #1.

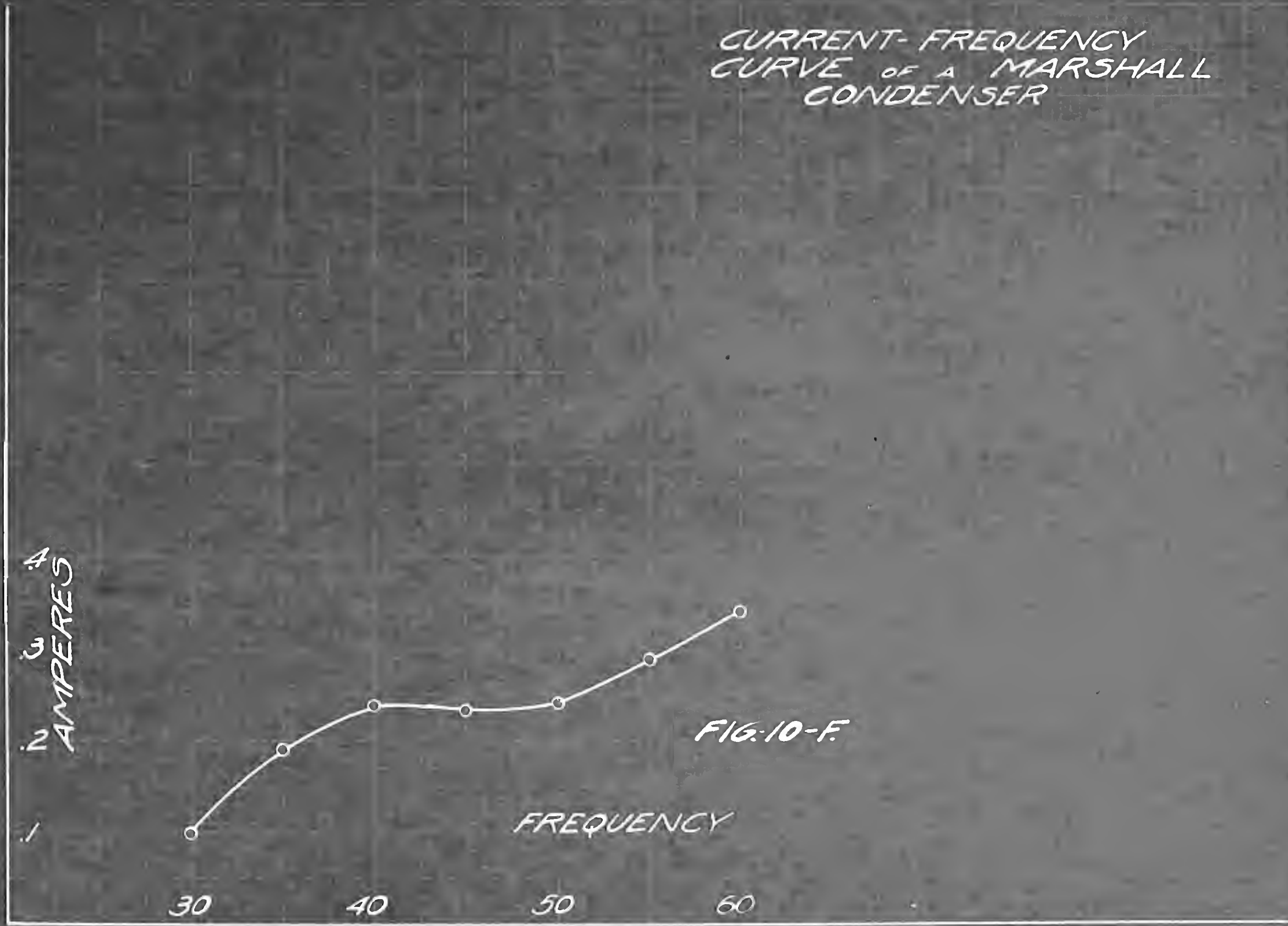
FIG. 10-E

condenser are listed below:

Current in amps.	Frequency in cycles	Const voltage
.34	60.0	
.29	55.0	
.24	50.0	
.23	45.0	
.24	42.5	
.24	40.0	
.19	35.0	
.10	30.0	

The curve showing the frequency current relation is sketched in fig. 10-F.

The data taken on condenser No. 3 (the telephone condenser) without any inductance are as follows:


CURRENT- FREQUENCY
CURVE OF A MARSHALL
CONDENSER
AMPERES
.1
.2
3
.4
FREQUENCY
30
40
50
60
FIG. 10-F.

Current in amps.	Frequency in cycles	Constant Voltage
0.818	30.0	
1.010	32.5	
1.125	35.0	
1.130	36.0	
1.080	37.5	
1.040	40.0	
1.021	42.5	
1.020	45.0	
1.040	47.5	
1.078	50.0	
1.180	55.0	
1.300	60.0	

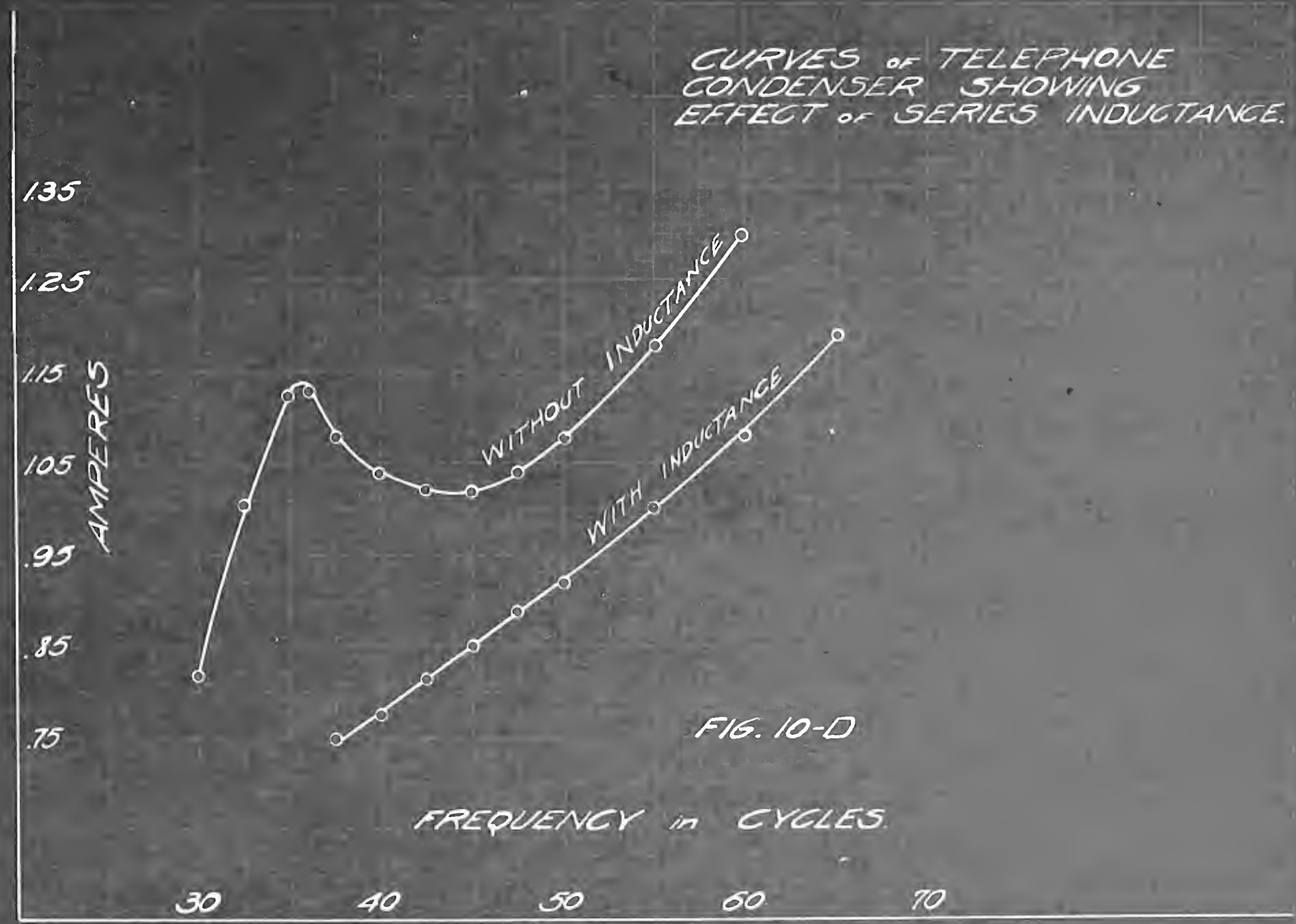
The data taken with inductance in series to damp out the harmonics are as follows:

Current in amps.	Frequency in cycles
.750	37.5
.775	40.0
.816	42.5
.850	45.0
.890	47.5

.920	50.0
1.004	55.0
1.080	60.0
1.190	65.0

The curves showing the relation between the current and frequency with and without a series inductance in circuit are drawn in Fig. 10-D. By a special connection of the alternator it was possible to secure a harmonic of the third or ninth harmonic. A study of the voltage wave of this generator showed a strong eleventh and thirteenth harmonic, and so in order to get a harmonic near to one of these the ninth was used. This gave approximately a sine wave and a frequency of nine times the fundamental.

In order to produce a ninth harmonic alone it was necessary to connect the alternator in delta and open one corner and then to form a resonance circuit for the ninth harmonic. The connections for the ninth harmonic are shown in fig. 10-G. In order to tell when



CURVES OF TELEPHONE CONDENSER SHOWING EFFECT OF SERIES INDUCTANCE.
1.35
1.25
1.15
1.05
.95
.85
.75
AMPERES
WITHOUT INDUCTANCE
WITH INDUCTANCE
FIG. 10-D
FREQUENCY in CYCLES.
30
40
50
60
70

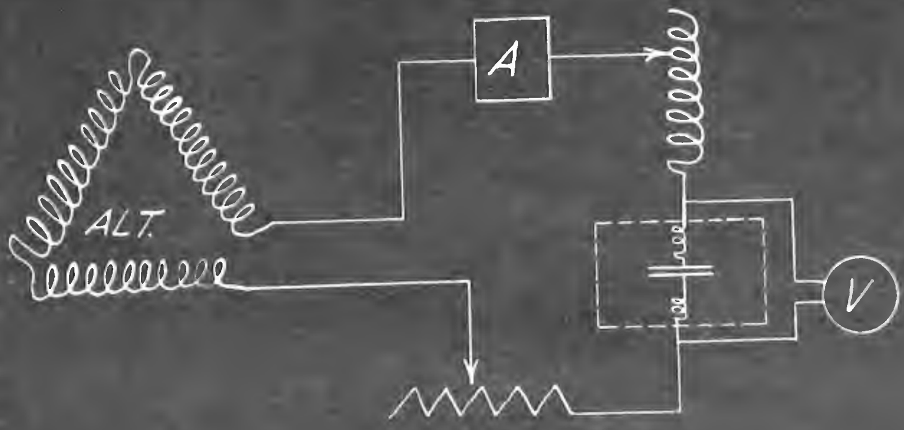


FIG. 10-G.

the circuit was in resonance for the ninth harmonic it was necessary to regulate the inductance until maximum deflection on the ammeter was obtained. A check on the frequency was obtained as follows: From the formula $I = 2 \Pi f C E$, the capacity of the condenser was calculated from the previous data and found to be 26.35 m.f.; the frequency of the ninth harmonic at the required point on the curve or 475 cycles was then substituted in the formula and the current required was 8.65 amps. When this amount was tabulated on the ammeter the frequency was very near to 475 cycles. The data taken for the ninth harmonic are as follows:

Current in amps.	Frequency in cycles
3.25	373.5
3.70	427.5
4.16	450.0
4.38	472.5
4.30	495.0
4.62	540.0

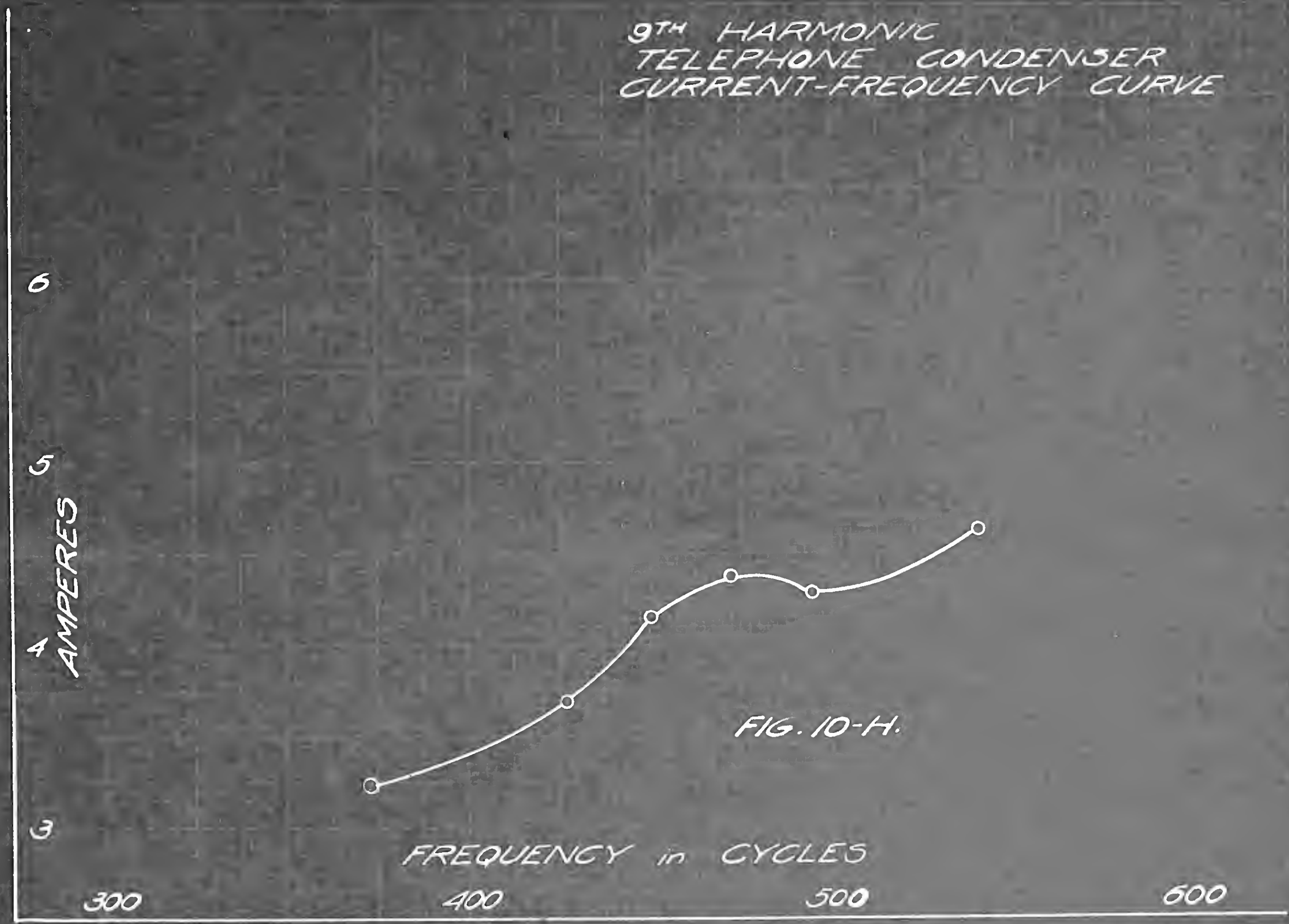

9TH HARMONIC
TELEPHONE CONDENSER
CURRENT-FREQUENCY CURVE
AMPERES
6
5
4
3
FREQUENCY in CYCLES
300
400
500
600

FIG. 10-H.

CONCLUSION

Before starting the experiment we assumed that the e.m.f. wave of the alternator which was used to test the condenser was nearly a sine wave.

This, however, was not true as it was found later. The actual e.m.f. wave of the machine was traced on an oscillogram, and it was further analyzed into its component parts by Runge's method. (See experiment No. 4. Analysis of wave forms equation (4).

This final equation indicates that the wave is very much distorted with pronounced 7th, 11th, and 13th harmonics

This was also proved by connecting the machine Delta, opening one corner, and connecting the condenser in series.

Resonance was obtained with machine frequency of 60°, and the frequency of the circuit was calculated, and found to be 423 cylces. This shows the resonance was due to the 7th harmonic which as we mention before is the most prominent.

It is therefore due to a-non-sine wave of e.m.f., and the various frequencies which enter whenever a resonance is established and also due to the fact that the condenser was wound inductively, that we observed the peculiar results, and the apparently abnormal behavior of the telephone condenser.

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